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Allan Ferguson.

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XIII. *Extra-Galactic Radio-Frequency Radiation*

By R. HANBURY BROWN and C. HAZARD

Jodrell Bank Experimental Station, University of Manchester*

[Received November 4, 1951]

SUMMARY

It has been shown previously that radio-frequency radiation is being received on the earth from the extra-galactic nebula N.G.C. 224 (M. 31). This paper reports some initial attempts to extend the measurements of extra-galactic radio radiation on a wavelength of 1.89 m. A radio survey has been made of three bright nebulae (in addition to N.G.C. 224) and three radio sources have been associated with them. The radio intensity to be expected from eight of the major clusters of nebulae has been calculated and two of the clusters have been surveyed and identified with radio sources. For a selected region of sky an attempt has been made to predict the irregularities in the radio isophotes corresponding to large scale irregularities in the distribution of extra-galactic nebulae. These predictions have been compared with radio observations of the region, and it is shown that there is a correlation between the radio isophotes and the distribution of extra-galactic nebulae. A preliminary examination has been made of the relation between the apparent photographic magnitude and the apparent radio magnitude of nebulae. The results suggest that there is a definite relationship and that the apparent photographic magnitude increases more rapidly than the apparent radio magnitude.

§1. INTRODUCTION

SINCE the original experiments of Jansky (1931) it has been recognized that radio-frequency radiation is being received on the earth from extraterrestrial sources. More recent measurements have shown that the isophotes of this radiation correspond well with the general outlines of the Galaxy as derived from visual observations. The existence of discrete sources of radiation, or radio stars, has been established since 1948 (Bolton and Stanley 1948, Ryle and Smith 1948), and a recent survey of the northern hemisphere (Ryle, Smith and Elsmore 1950) gives the position of 50 radio stars.

The results available at present suggest that the majority of the radio radiation received has its origin in the radio stars, and that the radio stars themselves are widespread throughout the Galaxy with a distribution, and even perhaps a density, similar to that of the more

* Communicated by Professor A. C. B. Lovell.

common type of star. Attempts to identify these radio stars with known visual objects have so far failed and it appears that the radio star may represent a new type of astronomical body.

In a recent work (Hanbury Brown and Hazard 1950) the authors have reported the detection of a radio source which has been identified with the great spiral nebula in Andromeda (N.G.C. 244, M. 31). This result suggests that, whatever the mechanism may be, radio-frequency radiation is to be expected from other extra-galactic nebulae.

The present paper reports some initial attempts to extend the measurements of extra-galactic radio-frequency radiation. The experiments reported here have been concentrated on three main lines of attack. Firstly, a survey has been made of the radio emission from four bright nebulae which lie within the field of the apparatus. Secondly, the results from this experiment have been used to predict the radio intensity to be expected from the major clusters of nebulae, and these predictions have been tested experimentally on two clusters which are conveniently within the view of the apparatus. Thirdly, an attempt has been made to detect irregularities in the radio isophotes corresponding to the large scale irregularities in the distribution of extra-galactic nebulae.

§2. DESCRIPTION OF APPARATUS

The apparatus used in these experiments has been described in detail elsewhere (Hanbury Brown and Hazard 1951), and only a brief description will be given here. The aerial system is a paraboloid of 218 ft. diameter and 126 ft. focal length and at a wavelength of 1.89 m the beam width is 2 degrees between half power points. The beam can be directed to declinations between N 38° and N 68° by tilting the mast which supports the primary feed.

The design of the receiving equipment is based on that described by Ryle and Vonberg (1948) in which the power received by the aerial is balanced continuously against that from a standard generator. The receiver is tuned to 1.89 metres and has a pre-detector bandwidth of 2 mc/s, a post-detector time-constant of 10 seconds and a noise factor of 3.5. The calibration of the whole equipment for the measurement of power depends upon the aerial gain and upon the standard generator which is a tungsten filament diode type CV. 172.

During the daytime the minimum flux which can be detected by this equipment is limited by various forms of interference; however, at night it has been found possible to approach the theoretical limits set by the fluctuations in the receiver. On favourable nights it is possible to detect on a single record a source whose intensity is about 10^{-25} watts/sq.m/c.p.s. and by averaging the results of at least four good observations this limit has been extended to 5×10^{-26} watts/sq.m/c.p.s. Most of the results given in this paper were obtained from records made between midnight and sunrise.

§3. METHOD OF OBSERVATIONS

The technique of observation was determined by the mechanical difficulty of moving the aerial beam rapidly. For the examination of a particular region of the sky the aerial beam was fixed at a constant elevation, corresponding to the required declination, and at a constant azimuth (approximately due south or due north). With the beam at this fixed elevation the power received was recorded for the required periods of right ascension. The procedure was repeated until an adequate number of good records was made. The measurements were then repeated with the beam at different elevations until the particular region of the sky had been surveyed.

§4. THE DETECTION OF INDIVIDUAL EXTRA-GALACTIC NEBULAE

Following the work on N.G.C. 224 (M. 31) a survey was made of the region around three of the bright nebulae in the field of view of the aerial. These are N.G.C. 5457 (M. 101), N.G.C. 5194 (M. 51) and N.G.C. 4258. In each case a radio source was found close to the coordinates of the nebulae. The results are shown in table 1 together with the previously published data on N.G.C. 224 (Hanbury Brown and Hazard 1951).

The presence of a source, probably a radio star, separated from the position of N.G.C. 5457 by about 2 degrees made the measurements of the weak source at the position of the nebula unreliable, and it was only possible to establish an upper limit to its intensity.

The identification of the two radio sources with the two nebulae N.G.C. 5194 and N.G.C. 4258 can only be made tentatively. The only two methods at present available for testing the identification are the comparison of the celestial coordinates of the sources with those of the nebulae, and the comparison of their intensities with those predicted from the other data. It is not possible with the present apparatus to measure the apparent diameter of these sources and to correlate them with the visual diameters of the nebulae, as was done in the case of N.G.C. 224. A comparison of the celestial coordinates shows that the positions of the two sources agree with those of the two nebulae within the limits of error of the measurements. It is possible, however, that the two sources are not associated with the nebulae and are 'foreground' radio stars which are fortuitously coincident in position with the nebulae. Ryle, Smith and Elsmore (1950) have published a survey of the radio stars in the northern hemisphere, and their data has been used to estimate the probability that either of these sources is a radio star. It can be shown from their results that the probability of finding a radio star whose intensity is comparable (± 2 magnitudes) with that of one of the sources, and which lies within the limits of error of its position (about ± 1 degree) is about 1 in 50. The probability that both sources represent accidental coincidences with radio stars is therefore very small.

Table 1. Radio Sources Corresponding to Individual Extra-Galactic Nebulae

Nebula	Coordinates of nebula. Epoch 1950	Apparent Photographic magnitude	Type of nebula	Coordinates of radio source. Epoch 1950	Intensity* of radio source (watts/sq.m/c.p.s.)	Apparent† radio magnitude of source
N.G.C. 224 (M. 31)	R.A. 00 ^h 40 ^m Decl. N 41° 00'	+5.0	Sb	R.A. 00 ^h 40 ^m 15 ^s ± 30 ^s Decl. N 40° 50' ± 20'	10 ⁻²⁴ ± 25%	+6.6
N.G.C. 5457 (M. 101)	R.A. 14 ^h 01 ^m 24 ^s Decl. N 54° 35'	+9.0	Sc	No reliable data, cannot be resolved from nearby source	< 10 ⁻²⁵	> +9.1
{ N.G.C. 5194 N.G.C. 5195 (M. 51)	R.A. 13 ^h 27 ^m 48 ^s Decl. N 47° 27'	+9.7	Sc	R.A. 13 ^h 27 ^m 30 ^s ± 2 ^m Decl. between N 46° and N 48° 30'	5.5 × 10 ⁻²⁶ ± 30%	+9.7
N.G.C. 4258	R.A. 12 ^h 16 ^m 30 ^s Decl. N 47° 35'	+10.2	Sb	R.A. 12 ^h 15 ^m 00 ^s ± 5 ^m Decl. between N 46° 30' and N 48° 30'	5 × 10 ⁻²⁶ ± 50%	+9.8

* The intensity given is double that measured in one plane of polarization.

† The scale of radio magnitude has been defined so that for (N.G.C. 5194 + N.G.C. 5195) the apparent radio and photographic magnitudes are numerically equal. The intensities given for N.G.C. 224 and (N.G.C. 5194 + N.G.C. 5195) are the most reliable in the table, and the latter has been arbitrarily selected as a standard.

The radio intensity to be expected from the two nebulae can be predicted from the measurements made on N.G.C. 224 by assuming that the ratio of the light flux to the radio flux is constant for all nebulae. Inspection of table 1 shows that the two radio sources are of about the same intensity as would be expected from the almost equal photographic magnitudes of the two nebulae; however, the observed value of their radio intensity is 2 magnitudes more intense in relation to their photographic magnitude than would be expected from the measured properties of N.G.C. 224. This discrepancy cannot be explained by known errors in the measurements, and is discussed in § 7.

§5. CLUSTERS OF NEBULAE

The intensity of the radio-frequency radiation received from the nebulae given in table 1 suggests that it should be possible to detect the radiation from the large clusters and groups of nebulae. The radio intensity to be expected from eight large clusters in the northern hemisphere has therefore been estimated by assuming that the ratio of radio flux to light flux found for N.G.C. 5194 applies to all nebulae. The results are expressed on the scale of radio magnitude defined in table 1. The integrated photographic magnitude of each cluster has been calculated from the number and the most frequent magnitude of the component nebulae as given by Hubble and Humason (1931). The frequency distribution of the magnitudes of the nebulae in each cluster has been assumed to be symmetrical about the most frequent magnitude, and the form of the distribution has been obtained from the data given for the Coma cluster. The estimated radio magnitudes of the eight clusters are given in table 2.

Zwicky (1937, 1951) has more recently suggested that the extensions of the clusters may be greater than estimated by Hubble and Humason. If these larger extensions are taken into account no maximum is observed in the luminosity function. The maximum found by Hubble and Humason is a selective effect due to a concentration of the brighter nebulae towards the centre of the cluster. Zwicky's results do not, however, seriously affect the figures given in table 2. An analysis of the Coma cluster shows that the concentration of nebulae, both in density and brightness, towards the centre of the cluster is such that radio radiation from the outer regions of the cluster would not be detectable, and of the nebulae in the central regions only the brighter magnitude groups are important. In the case of the Coma cluster, therefore, the results given by Hubble and Humason are satisfactory for the present estimate, and it will be assumed that this is also true of the other clusters.

Only four of the clusters given in table 2 lie within the working field of the present aerial system and, so far, only two of them have been investigated, namely, the Perseus cluster and the cluster centred on N.G.C. 911. In each case a radio-frequency source has been detected close to the position of the cluster, and the results of the measurements

Table 2. Calculated Radio Magnitudes of Eight Clusters of Nebulae

Cluster	Diameter of cluster (degrees)	Number of component nebulae	Most frequent apparent photographic magnitude	Predicted radio magnitude (scale as for table 1)
(1) Pegasus	1.0	100	+15.5	+10.1
(2) Cancer	1.0	150	+16.0	+10.1
(3) Perseus	2.0	500	+16.4	+9.2
(4) Coma	1.7	800	+17.0	+9.3
(5) Ursa Major I	0.7	300	+17.8	+11.2
(6) Ursa Major II	0.2	300	+18.7	+12.1
(7) Leo	0.6	200	+17.9	+11.7
(8) Cluster centred* on N.G.C. 911	1.0	—	—	<+11.5

* Dr. Harlow Shapley has kindly supplied the apparent photographic magnitude of the brightest 44 members of this cluster. The predicted radio magnitude was derived only for these 44 nebulae, together with the bright nebula N.G.C. 891 which lies on the edge of the cluster.

Table 3. Data on the Two Radio Sources Associated with Clusters of Nebulae

Cluster of nebulae	Coordinates of centre of cluster. Epoch 1950	Coordinates of radio-frequency source. Epoch 1950	Predicted radio magnitude of cluster (see table 2)	Observed radio magnitude of source.
Perseus	R.A. 3 ^h 16 ^m 15 ^s Decl. N 41° 19'	R.A. 3 ^h 15 ^m 15 ^s ± 1 ^m 30 ^s Decl. N 41° 22' ± 30'	+9.2	+7.2
Cluster centred on N.G.C. 911	R.A. 2 ^h 22 ^m 43 ^s Decl. N 41° 43'	R.A. 2 ^h 19 ^m ± 3 ^m Decl. 42° 10' ± 30'	<+11.5	+9.3

are shown in table 3. Table 3 also gives the coordinates and the predicted radio magnitudes of the two clusters. Inspection of the results shows that the celestial coordinates of the two sources agree closely with those of the corresponding clusters.

The observed radio magnitude of the source close to the Perseus cluster differs from the predicted radio magnitude of the cluster by 2 magnitudes, while the radio magnitude of the second source differs by less than $2\frac{1}{2}$ magnitudes from the predicted value. As the two clusters lie in low galactic latitudes, the apparent photographic magnitudes of the component nebulae may have been affected by visual obscuration in the Galaxy. If the obscuring matter reduces the intensity of the light received from the cluster without attenuating the radio flux, then the predicted radio magnitudes in table 3 will be too high. The visual obscuration for the Perseus cluster has been estimated by Hubble and Humason to be 0.4 magnitude on the basis of the abnormal colour excess of its component nebulae, and the predicted radio magnitude of this cluster may, therefore, be too great by this amount. The second cluster lies in the same region and its predicted radio magnitude may be subject to the same error. The discrepancy between the observed and the predicted radio intensity from the Perseus cluster suggests either that the radiation of a significant number of nebulae has been neglected or that the radio magnitudes of the component nebulae have been overestimated. For example, the results can be explained by assuming that a nebula of apparent photographic magnitude +16 has a radio magnitude of +14 (on the scale defined in table 1). It follows that all the predicted radio magnitudes in table 2 may be in error by the same amount as that predicted for the Perseus cluster, namely 2 magnitudes.

An attempt has been made to measure the apparent diameter of the two radio sources. In the case of the Perseus cluster the component nebulae lie in a group of diameter about 2 degrees, and it was hoped to measure the diameter of the associated radio source by observing the apparent broadening of the aerial beam as it swept across the cluster. The degree of broadening to be expected has been calculated for the case of the beam passing over the Coma cluster. This particular cluster was used in the calculation because the apparent angular distribution of its component nebulae has been published in detail (Zwicky 1937) and because it is of comparable angular diameter to that in Perseus. The calculations showed that the maximum degree of broadening of the beam, measured between half-power points, would only be about 25% even if the concentration of the brighter nebulae to the centre of the cluster is neglected. It is to be expected that the broadening would be much less in practice due to this concentration of the brighter nebulae. The observed results show that the apparent broadening of the beam is less than 25%, but the records are not sufficiently good to permit a more accurate measurement. Hence the measurement of the apparent angular diameter of these sources must await more refined measurements, either

with a narrower beam or with an interferometer. It should also be noted that the value of intensity given for the sources is not a true integrated value since it has been assumed that they are point sources.

It is possible that the sources are, in fact, radio stars in the Galaxy and that their positions happen to coincide by chance with those of the clusters. From the data given by Ryle, Smith and Elsmore (1950), it may be shown that the chance that a radio star falls within the limits of error of the position given for the source near the Perseus cluster is about 1 in 500. For the weaker source (near N.G.C. 911) this chance is about 1 in 100. The probability that both sources represent accidental coincidences is therefore very low. From the close agreement of the celestial coordinates given in table 3, and from the reasonable agreement between the predicted and the observed intensities, the two radio sources may be tentatively identified with the clusters.

§6. LARGE SCALE IRREGULARITIES IN THE DISTRIBUTION OF EXTRA-GALACTIC NEBULAE

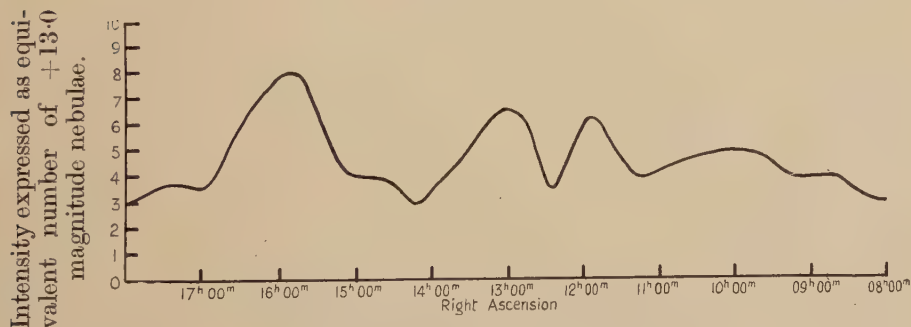
An attempt has also been made to detect by radio-frequency radiation the large scale irregularities in the distribution of extra-galactic nebulae. An examination of the distribution of the bright nebulae, as given by Shapley and Ames (1932) shows conspicuous grouping in high latitudes in the northern galactic hemisphere. Since this grouping is observed in a region where the effects of visual obscuration by the Galaxy are small, it appears likely that it must correspond to a genuine large scale clustering of the nebulae. It is therefore to be expected that these irregularities in the distribution of the nebulae will give rise to irregularities in the radio isophotes of the corresponding regions of the sky.

(a) *Analysis of the astronomical data on the distribution of nebulae*

For the purpose of a preliminary investigation a region of sky was chosen between declinations $N 44^\circ$ and $N 55^\circ$ and right ascensions 10^h and 18^h . This region contains marked grouping in the visual distribution of nebulae, and it also lies largely in high galactic latitudes where the disturbing effects on the observations due to the radiation from the Galaxy are at a minimum. The main features of the distribution of nebulae in this region have been found by considering only the nebulae fainter than magnitude $+13.0$. In this preliminary analysis the effect of the bright nebulae has been omitted since they are so scattered that they must be considered individually for each position of the aerial beam. Their effect has, however, been taken into account when making the detailed investigation of the radiation received at declination $N 47^\circ 30'$ as given in § 6 (c). The analysis has been based on a detailed survey made by Shapley and Jones (1940) which gives the counts of nebulae on a series of photographic plates taken in a strip 5 degrees wide in declination and centred on $N 43^\circ$. These counts cover the nebulae in a magnitude range down to $+17.5$ and are divided into

groups at intervals of 0.1 magnitude. The results have been used to estimate the variation with right ascension of the radio intensity which would be received from these nebulae by a 2 degree beam as it sweeps round a line of constant declination centred in this strip. In making this estimate the intensity from the nebulae in the beam at every value of right ascension was found by weighting the contribution from the nebulae according to their photographic magnitudes, and by expressing the sum of their radiation in terms of an equivalent number of +13.0 magnitude nebulae. The number of nebulae in the beam at each position was found from the effective coverage of the beam and from the average density of nebulae on each photographic plate. The results of the analysis are given in fig. 1 and show clearly the irregularities in nebular distribution which can also be seen on the maps of bright nebulae. There are concentrations of nebulae to be seen at approximately R.A. 12^h, R.A. 13^h and R.A. 16^h, and the maps of the distribution of bright nebulae suggest that these concentrations may extend over a considerable range of declination at about the same values of right ascension.

Fig. 1



Calculated variation with right ascension of the radio intensity received from nebulae in a magnitude range +13.0 to +17.5. Declination N43°.

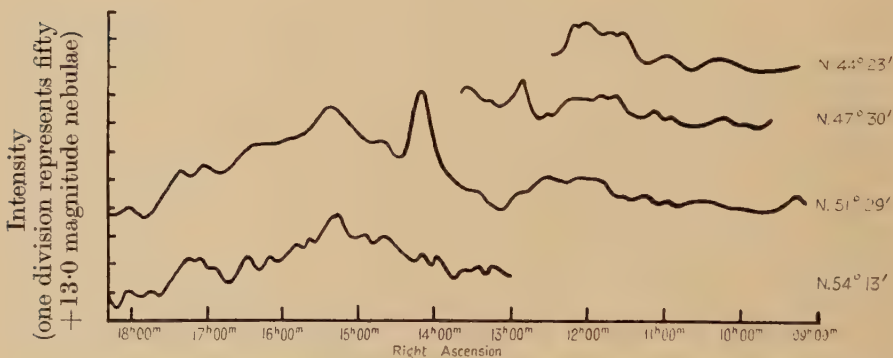
The intensity of the corresponding irregularities to be expected in the radio isophotes was estimated by using the same relation between the radio magnitude and the photographic magnitude as in § 5. It can be seen from fig. 1 that the maximum irregularity in the distribution of nebulae in the beam corresponds to the intensity from about four +13.0 magnitude nebulae, and hence to a radio magnitude of +11.5. The limiting sensitivity of the present equipment corresponds to a radio magnitude of about +10, and hence the maximum irregularity shown in fig. 1 will be 1.5 magnitudes below the limits of detection. Since, however, the survey extends only to nebulae of magnitude +17.5, it is not possible to predict accurately the magnitude of the corresponding irregularities in the radio isophotes as the tendency to cluster clearly extends to fainter nebulae. However, the curve in fig. 1 suggests that,

if the total contribution from the nebulae outside the range of magnitude $+13$ to $+17.5$ is appreciable, then it might be possible to detect the irregularities in the radio isophotes with the present equipment.

(b) *General comparison of the radio observations with the astronomical data*

A preliminary survey has been made of the radio radiation from the selected region. The survey consists of a series of curves showing the radio intensity versus right ascension observed at a number of different declinations. These observed curves all show small irregularities superimposed on a gradient of the 'foreground' radiation from the Galaxy, which even at high galactic latitudes is considerable. In order to show these irregularities on a suitable scale, it has been necessary to subtract from the observed curves an arbitrary smooth gradient. The exact shape of this gradient is not important since it has been taken as very smooth compared with the irregularities. The resultant curves are shown in fig. 2, and they show the presence of two broad maxima at

Fig. 2



Irregularities in the background radiation observed at different declinations.

R.A. 12^h and R.A. 15^h to 16^h . (The two sharp maxima at about R.A. 14^h and R.A. 13^h are probably radio stars.) These broad maxima are of very low intensity, and to show them clearly above the random noise it has been necessary to average at least four records at each value of declination. It can be seen from fig. 2 that the maxima persists over a range of declination, and this suggests the presence of two extended bands of radiation. The correlation between the position of these bands and those predicted from the survey of nebulae suggests that the irregularities in the radio intensity are associated with the clouds of nebulae and that they do not represent effects due to the radiation from the Galaxy.

The intensity of the irregularities is much greater than predicted in fig. 1, and the discrepancy between the predicted and observed values varies from 3 to 4 magnitudes.

(c) *Detailed comparison of the radio observations with the astronomical data for declination N 47° 30'*

A more detailed effort has been made to correlate the distribution of nebulae with the shape of the radio irregularities observed when the beam is directed to declination N 47° 30'. In this analysis the additional effect of the bright nebulae was taken into account by considering them individually. The radiation from each nebula was weighted according to its photographic magnitude and its position relative to the beam when centred on declination N 47° 30'. The sum of the radiation from the bright nebulae in the beam was then found for all values of right ascension, and was combined with the results given in fig. 1 to give the contribution from all nebulae down to magnitude +17.5. The resultant curve is shown in fig. 3 (a). This procedure assumes that the irregularities in the distribution of faint nebulae (magnitude +13.0 to +17.5) found for the strip between declinations N 41° to N 46° can be applied to a strip 2 degrees wide centred on declination N 47° 30'.

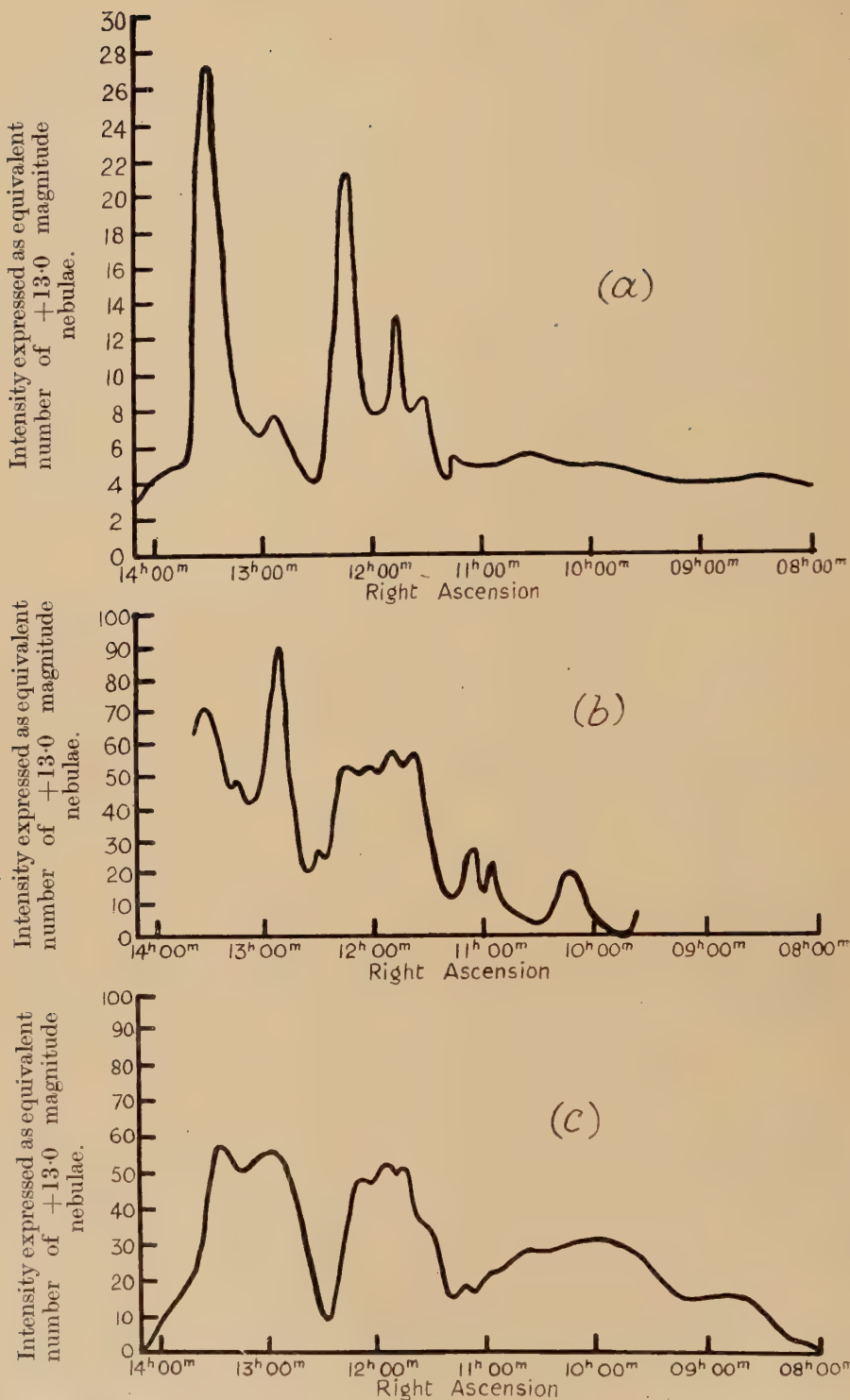
Fig. 3 (b) shows the irregularities observed in the radio isophotes over a short period of right ascension in a strip centred on declination N 47° 30'. The curves in fig. 3 (a) and fig. 3 (b) both show a broad maximum at R.A. 12^h; however, the effects of individual bright nebulae, which are so marked in fig. 3 (a), are not clearly shown in the observed curve for fig. 3 (b). It can also be seen that there is a discrepancy between the predicted and the observed intensity of about 3 magnitudes. This disagreement is not unexpected since the contribution from nebulae fainter than +17.5 has been omitted. It is, however, interesting to find that if the contribution from the fainter nebulae is increased by a factor of fifteen and is then added to the contribution from the bright nebulae, a much closer agreement in shape can be obtained. This is shown in fig. 3 (c).

The sharp peak in the observed curve of fig. 3 (b) at R.A. 12^h 50^m is probably due to a radio star, but in view of the results presented in this paper the possibility that it is a cluster of nebulae cannot be excluded.

§7. DISCUSSION OF RESULTS

The results given in §§4 and 5 have been examined for any relationship between the apparent photographic and the apparent radio magnitudes of the nebulae observed. The available data is scanty and is summarized in table 4. In fig. 4 the difference between the two scales of apparent magnitude is plotted against the apparent photographic magnitude. The value given for a +16.0 magnitude nebula, derived from the Perseus cluster must be regarded as uncertain; however, the results suggest that there is a relationship between the two magnitude scales, and that the apparent photographic magnitude increases more rapidly than the apparent radio magnitude.

Fig. 3



Comparison of the calculated and observed irregularities in the background radiation for a strip centred on declination $N 47^{\circ} 30'$.

- (a) Calculated curve for all nebulae down to magnitude $+17.5$.
- (b) Observed curve.
- (c) Calculated curve after increasing the contribution from nebulae fainter than magnitude $+13.0$ by a factor of fifteen.

Table 4. Comparison of Apparent Photographic and Radio Magnitudes of Extra-Galactic Nebulae

Nebula	Type of nebula	Apparent photographic magnitude (M_p)	Apparent radio magnitude (M_r)	$M_r - M_p$
N.G.C. 224	Sb	+5	+6.6	+1.6
N.G.C. 5457	Sc	+9	> +9.1	> +0.1
{ N.G.C. 5194 N.G.C. 5195	Sc	+9.7	+9.7	0.0
N.G.C. 4258	Sb	+10.2	+9.8	-0.4
Value derived* from an analysis of the Perseus cluster	Mainly type E	+16.0	+14.3	-1.7

* In deriving the apparent radio magnitude given in this table, the photographic magnitudes of the nebulae in the Perseus cluster have been decreased by 0.4 magnitude to allow for visual obscuration (see text, §5).

A possible explanation of this discrepancy between the two scales might be found in the difference between the types of nebulae observed; however, this seems to be unlikely. Both N.G.C. 4258 and N.G.C. 224 are Sb type nebulae and yet they show the discrepancy in the two scales. Furthermore, the majority of the nebulae in the Perseus cluster are type E nebulae, and the comparatively high intensity observed from this cluster would have to be explained by assuming that the ratio of radio flux to light flux is greater for type E than for types Sb or Sc. If this assumption were correct then the radio radiation from the bright elliptical companion (N.G.C. 205) of N.G.C. 224 would be comparable to, or greater than, the radio radiation from N.G.C. 224 itself, and the measurements on N.G.C. 224 indicate that this is not so. It is clear that at present there are insufficient experimental results to establish the relationship between the scales of radio and photographic magnitude, but the present preliminary measurements suggest a divergence between the scales which requires further investigation.

If the relationship between the photographic and the radio magnitudes suggested by fig. 4 holds for all nebulae, even approximately, then it is unlikely that any known individual extra-galactic nebula will be observed as a radio source more intense than N.G.C. 224. Many of the known radio sources are in fact much brighter than N.G.C. 224, and the absence of correlation between the positions of these sources and those of visual objects, together with their small apparent angular diameter (less than a few minutes of arc) supports the view that they cannot be identified with individual extra-galactic nebulae. This evidence, together with the observed concentration of the bright sources towards the galactic plane (Westerhout and Oort 1951), suggests that the brighter of the known sources are in the Galaxy. The results of a survey of the northern hemisphere (Ryle, Smith and Elsmore 1950) indicate that there are at least 50 radio sources brighter than radio magnitude $+10$ (as defined in table 1), and a survey of the bright external galaxies shows that in this region there are only 10 nebulae with an apparent photographic magnitude, and hence an apparent radio magnitude, less than $+10$. It therefore seems probable that less than 20% of the sources known at present are individual extra-galactic nebulae.

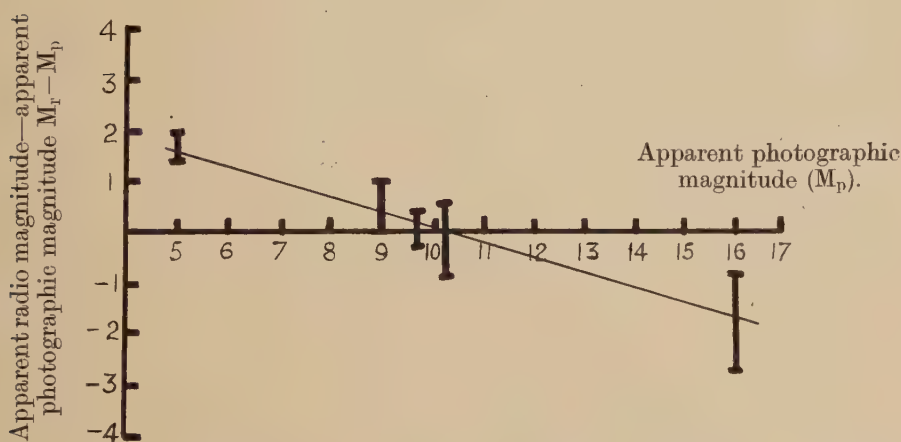
The extension of the work to fainter individual extra-galactic nebulae presents considerable difficulties, as not only will it be difficult to detect such weak sources, especially in the presence of irregularities in the radiation from the Galaxy, but it will also be increasingly difficult to distinguish nebulae from radio stars. On the other hand, the detection of clusters of nebulae promises a method of extending the radio observations to much fainter nebulae. It appears from the results in § 5 that there are several clusters of nebulae whose radio radiation should be detectable as weak sources, and it may prove that these clusters can be distinguished from radio stars by their apparent angular diameter. A study of these clusters should yield information on the radio radiation

from faint nebulae, and it may also give data on the relative radio intensity from different types of nebula.

The observations of the large scale irregularities in the distribution of extra-galactic nebulae are not yet sufficiently advanced to warrant a detailed discussion. However, the preliminary results suggest that the analysis of extra-galactic radio-frequency radiation may provide a new tool for examining the metagalaxy.

In a previous publication (Hanbury Brown and Hazard 1951) the authors estimated that the contribution of all extra-galactic nebulae to the radio-frequency radiation incident on the earth is about 1% of the total radiation observed. This estimate was made by applying to all nebulae the ratio of radio flux to light flux measured for N.G.C. 224. The results

Fig. 4



Comparison of apparent radio and photographic magnitudes of nebulae. The full line represents only the slope of the relationship between M_p and $M_r - M_p$, and to give the same numerical values it should be displaced to pass through the point $M_p = 9.7$.

shown in fig. 4 suggest that this ratio cannot be applied to all nebulae and that the magnitude of the extra-galactic radiation has been underestimated. If the calculation is corrected by means of the data in fig. 4 then the value of the extra-galactic contribution rises to approximately 10%. This corresponds to an effective aerial temperature, averaged over the whole sky, of about 100° K at 100 mc/s. In a recent paper Westerhout and Oort (1951) find that the contours of radio-frequency radiation, observed over the whole sky, can be explained in terms of radiation from the Galaxy superimposed on an isotropic contribution of 600° K at 100 mc/s. They suggest that this temperature may represent the radiation from extra-galactic nebulae and it is interesting to note that the results presented in this paper also suggest that there should be a significant extra-galactic contribution.

ACKNOWLEDGMENTS

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XIV. *The Distribution of Electrons Round Impurities in Monovalent Metals*

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SUMMARY

A theoretical investigation is made of the distribution of electrons round a positively charged impurity dissolved in a monovalent metal. Applications are made to dissolved hydrogen, where the impurity is a proton, and to atoms such as zinc, gallium, etc., which are usually considered to contribute their electrons to the conduction electrons. In all cases the positive charge must be screened; and in many cases this is shown to occur through the formation of bound states below the level of the Fermi distribution. The relation of these results to the Hume-Rothery rule is discussed. The ideas introduced are used to calculate the heats of solution of hydrogen and of polyvalent metals in the noble metals, and to discuss the magnetic properties of these alloys. A detailed discussion is given of x-ray emission and absorption, the vacancy left in the x-ray shell being here treated as the positive impurity. In certain cases quantitative predictions are made about the energies of x-ray absorption edges.

Discussions along the same lines are given of the optical absorption of the noble metals, and of the x-ray emission spectra of certain alloys.

§1. INTRODUCTION

THE primary purpose of this paper is to study the distribution of electrons in metals such as copper, silver and gold in the neighbourhood of substitutional impurities of different valency such as Ni, Zn, Ga, Sn, As. The Hume-Rothery rule suggests that the last four of these metals lose respectively one, two, three and four electrons to the conduction band; on the other hand the electrical resistance (Mott 1936) and other evidence suggest that there must be a concentration of negative charge round the dissolved ion, so that a volume surrounding it is neutral. Similarly around Ni or around a vacant site there will be a concentration of positive charge. In this paper, we study the nature of this polarization, or concentration of charge, and its effect on the binding energies and magnetic properties of alloys, and on the x-ray emission and absorption spectra of metals and alloys.

* Communicated by Professor N. F. Mott, F.R.S.

The problem is treated throughout by the Hartree-Fock approximation, in which its own wave function is ascribed to each electron. The main problem of interest is the following : in the case of attractive potentials (e.g. those of Zn, Ga, etc. in Cu, or of a proton in a Fermi gas) do there or do there not exist occupied bound levels below the lowest level of the Fermi distribution ? It should be pointed out that this is not merely a question of finding the most accurate approximation for calculating energies, but is capable of direct experimental investigation by x-ray emission or magnetic measurements.

So far, most of the calculations dealing with dilute alloys have been concerned with two atoms of the same valency. Thus Huang (1948) has dealt with Au in Ag and Arafat (1949) with Cu in Ag and Ag in Cu. Huang's calculations, none the less, show a polarization round the impurity. Huntington's (1942) calculations for the energy of a vacant lattice point in copper deals with an impurity having negative charge (like nickel), so that the question of bound states does not arise. Mott (1936, Mott and Jones 1936 a) applies the Thomas Fermi method to the case of positively charged impurities ; but this method implicitly assumes the existence of bound states, and so begs the question.

Our general conclusion is as follows. For an impurity in a monovalent metal carrying an excess charge of $2e$, $4e$ etc. (Ga or As in Cu) there will *always* exist doubly occupied bound states below the level of the Fermi distribution. These are subtracted from the first Brillouin zone, so that the total number of electronic states remains the same as before ; Jones's (1934) explanation of Hume-Rothery's rule is thus not affected. The screening of the positive charge is entirely due to these electrons. The Fermi electrons are affected as follows : there will be an absence of charge at the centre of the impurity, compensated by a heaping up of charge in a region round it ; the total displacement of charge must be zero, so no contribution is made to the screening.

In the case of an excess charge unity, as for example Zn in Cu, an occupied level may or may not exist. Three configurations are possible, depending on the relative energies of this level and of the conduction band. If no occupied level exists, screening is produced by a shift of the conduction electrons, in the same way as for a negative charge (this is probably so for Zn or H in Cu etc.). If the bound level is occupied by one electron, this screens the charge, and the conduction electrons are polarized in such a way as to give no resultant shift of charge. Finally two electrons in the bound level would provide with the positive charge of the ion a net negative charge, which must be screened by pushing away the Fermi electrons. Configurations of the last two types appear in x-ray phenomena and probably when the alkali or alkaline earths metals are solvents of other metals. With an odd excess charge (e.g. Sn in Ag) similar results apply to the outermost electron of the dissolved monovalent ion (e.g. Sn^+). In other cases the additional electrons are always in bound states.

We use in this paper two different approaches. We first study the disturbances of a simple Fermi gas by a point charge (§§ 2 to 4). This illustrates well the principles involved. The study of bound states in real metals, our main purpose, is best made by estimating their energy relative to that of the Fermi electrons (§ 6). The approximations made are justified by the analysis of a simple case, hydrogen in copper (§ 7). Applications are made to energies of solution (§ 9), magnetism (§ 10) and x-ray spectra (§ 11).

§ 2. BEHAVIOUR OF AN ELECTRON IN A GIVEN FIELD OF FORCE

Before studying the conduction band of a metal, we must obtain some formulae about the motion of an electron in a spherical field of force: conditions for the existence of bound states and the displacement of charge towards the centre will be investigated.

Let then an electron be enclosed in a sphere of large radius R and move in a spherically symmetrical field of potential $V(r)$. If $V(r)$ tends to zero more rapidly than r^{-2} at infinity,* the possible electronic wave functions have radial parts with asymptotic form (see for instance Mott and Massey 1949 a):

$$\psi_l(r) \sim r^{-1} \sin(k_l' r + \eta_l - \frac{1}{2}l\pi) \quad (l=0, 1, 2 \dots).$$

If we make the wave function vanish at the boundaries of the (large) sphere, we find the following quantized values of the momentum, valid for large R :

$$k_l' \sim (\lambda + \frac{1}{2}l)\pi/R - \eta_l/R$$

where λ is an integer. If k_l is the value of k_l' when V vanishes:

$$k_l \sim (\lambda + \frac{1}{2}l)\pi/R$$

$$\text{and} \quad k_l' - k_l \sim -\eta_l(k_l')/R. \quad \dots \dots \dots (1)$$

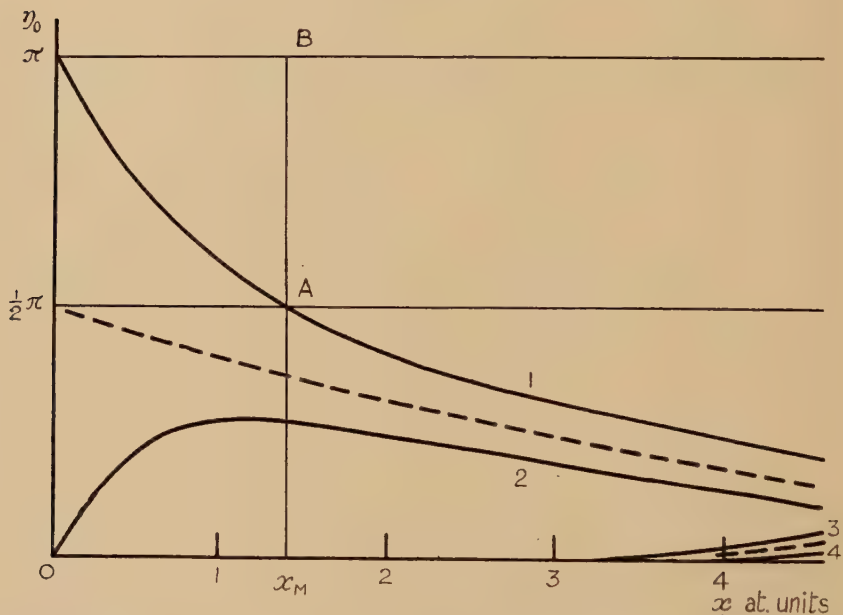
There is thus a one to one correspondence between quantized momenta k_l and k_l' , and the difference between them depends only on the phase.

We discuss now the dependence of η on k . η tends to zero for large k ; it will tend to one of the values $\lambda\pi$ as k tends to zero. λ is here an integer or zero: it is zero for repulsive or small attractive potentials, and increases with the strength of attractive potential V . This behaviour is shown in fig. 1. For limiting values of V just on the borderline between two values of λ , η_0 will tend to one of the values $\frac{1}{2}\pi, \frac{3}{2}\pi, \dots$ as shown by the dotted line in fig. 1. The other phases η_l do not behave in this way (cf. Mott and Massey 1949 c).

* For potentials $V(r)$ varying more slowly than r^{-2} , and as is well known for Coulomb fields (Mott and Massey 1949 b), η_0 cannot be defined for zero momentum. As seen later, this means for attractive potentials that there is an infinity of bound states which the Fermi electrons could occupy.

We study now the origin of the bound states. Let us assume for instance that there is one bound state for the potential V , hence that $\eta_0(0)=\pi$ (curve 1, fig. 1). Then, according to eqn. (1), the states $k_0=\pi/R, 2\pi/R \dots$ etc., correspond to the various states k'_0 in the presence of the field, $k_0=\pi/R$ corresponding to the lowest one $k'_0=0$ (fig. 2). Only the state $k_0=0$ cannot be related to any real value of k'_0 , but is replaced by the bound state, which may thus be considered as *subtracted from the bottom of the Fermi distribution*. It is easily verified that this result is general: the bound states for a particle in the potential V are always subtracted in that way from the lowest states in the Fermi distribution.

Fig. 1



Phase shifts $\eta_l(x)$ for attractive potentials (after Morse). x denotes $2kq^{-1}$.

1: $\beta=1.0, l=0$; 2: $\beta=0.7, l=0$; 3: $\beta=1.0, l=1$; 4: $\beta=0.7, l=1$.

We now suppose that there are N electrons in the sphere of radius R and assume that $V(r)$ is the self-consistent potential energy of each electron in the field of some impurity and of the Fermi gas thus formed. Let k_M be the maximum momentum before the impurity is introduced. We compute the electronic charge which must be added, when the impurity is present, to obtain the *same* maximum k_M .

According to eqn. (1), k_M corresponds in each component l to a different value k_{Ml}' of the momentum:

$$k_{Ml}' - k_M \sim -R^{-1} \eta_l(k_{Ml}'). \quad (1a)$$

We must therefore add electrons to occupy states in the interval between $k_{M'}$ and k_M (fig. 2). The number of such states, multiplying (1 a) by a density factor, is

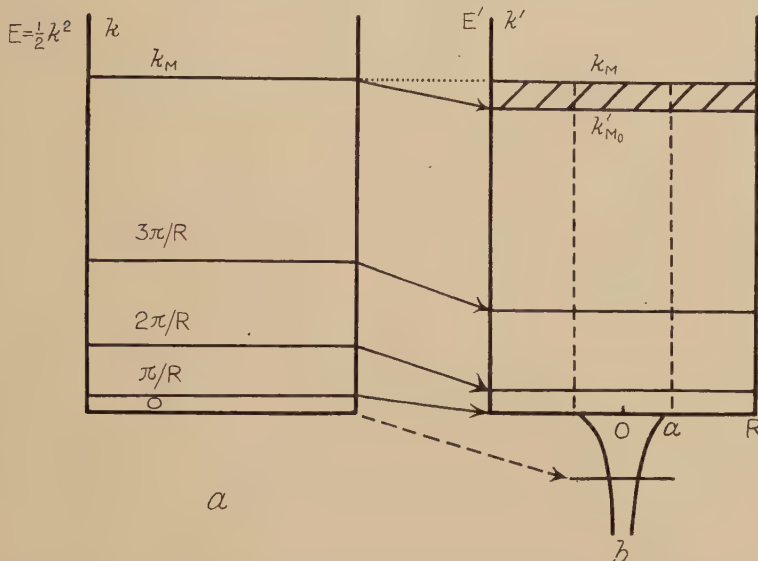
$$R\pi^{-1}(2l+1)(k_{M'}-k_M) \sim -(2l+1)\pi^{-1}\eta_l(k_M)$$

where we replaced $\eta_l(k_{M'})$ by $\eta_l(k_M)$ because η_l does not vary rapidly near k_M .

Summing over l and over both spins σ , the total charge introduced is therefore :

$$n = -\pi^{-1} \sum_{\sigma} \sum_l (2l+1) \eta_l(k_M). \quad (2)$$

Fig. 2



Displacement of states with s symmetry in a box of radius R , when an attractive potential V is introduced (a) without V ; (b) with V .

The quantity n can now easily be shown to be equal to the total excess charge *locally* attracted by the impurity. For let a be the radius where the potential V vanishes (fig. 2). It is a well known result of any consideration of normal modes of a vibrating medium, that, if $k_M(R-a)$ is large, the electronic density in the spherical shell $a \leq r \leq R$ is independent of the boundary conditions at $r=a$ and $r=R$, and the same as in an unperturbed Fermi gas (von Laue 1914). The total charge below k_M in the spherical shell is thus not altered when the impurity is introduced. The charge added, n , must therefore be localized in the sphere of radius a where the potential V is different from zero.

In eqn. (2), the bound states are counted as occupied. A differentiation gives

$$-\pi^{-1} [\partial \eta_l(k) / \partial k] dk$$

for the charge displaced by potential V in the l th component, for values of k in range dk . In other words, the attraction of charge is obtained by a compression of the states in momentum space. Appendix (1) gives another proof of this result.

The locally displaced charge n must compensate exactly the charge introduced by the impurity, so as to give no Coulombian term in $V(r)$. Usually, the impurities introduced in the physical problem are electrically *neutral* (H, Zn . . . atoms; vacancies . . .): if the centre is charged, electrons are introduced too. Thus the charge introduced in the gas is just equal to n , and the *Fermi gas conserves the same maximum momentum k_M* .

In other words, a neutral impurity introduced into a Fermi gas modifies the density of states in the k space in such a way as to keep constant the top of the Fermi band, at least for small concentrations where impurities do not interact. This point is of importance in computing the energies of solution (cf. § 7, 9).

Consider now as an example the screened Coulomb fields*

$$V = -Zr^{-1} \exp(-qr) \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

which will be used in the next section. Z and q are two constants, and the corresponding phase shifts η_l were computed numerically by Morse (1932) for positive values of Z . We reproduce in fig. 1 three examples of variations of $\eta_l(k)$. Morse's notation is

$$x = 2k q^{-1}, \quad \beta^2 = Zq^{-1}.$$

Contributions by p and higher components are negligible up to large momenta k , and the charge attracted is practically of pure s character. For small attractive potentials (e.g. $\beta = 0.7$), η_0 presents a maximum. Thus the density of attracted charge decreases with increasing momentum and some charge is even repelled for large momenta if the energy of the Fermi gas has a sufficient width.

For stronger attractive potentials (e.g. $\beta = 1.0$), a bound state appears. For all values of k and especially for small values, there is then a deficit of charge near the origin. The same occurs in the limiting case where $\eta_0(0) = \frac{1}{2}\pi$ ($\beta \sim 0.9$). η_0 has then a negative slope not only for potentials of form (3) but also for square potential walls of any dimensions,† hence probably for any kind of purely attractive potential V . This repulsion will be used in the next section.

In conclusion, we studied in this section the displacement of the quantized levels in an enclosed Fermi gas, when a potential $V(r)$ is introduced. The bound states appearing in an attractive potential are subtracted from the bottom of the Fermi band, and the charge locally attracted in the Fermi gas is simply related to the phase shifts of the quantized levels.

* Atomic units used throughout.

† For a square well of radius a , the slope of η_0 at the origin is $-\frac{1}{2}a$, independently of the depth of the well.

§3. SELF CONSISTENT SCREENING OF ADDITIONAL CHARGES BY FERMI ELECTRONS.

The discussion so far applies to a gas of electrons moving in an arbitrary field of potential $V(r)$. We shall study now the form of this field when it is due to an impurity carrying a point charge Z (e.g. $Z=+1$ for a proton). To the Coulomb term Zr^{-1} we must add a term due to the displacement of the electrons, obtained by a self consistent method. The displaced charge must exactly screen the charge Z ; it may be provided either by bound electrons having negative energy below the bottom of the Fermi distribution, or by electrons of positive energy and thus by the Fermi electrons themselves. We restrict ourselves here to the study of screening by Fermi electrons and show that for the simple Fermi gas, it occurs round negative charges Z only.

Using the Thomas-Fermi method, Mott (1936) obtained for the self consistent potential V round a charge Z in a Fermi gas the screened Coulomb field given above by formula (3). q is a constant nearly equal to 1 atomic unit. For copper for instance, Mott's computation gives $q=0.98$. Isenberg's Thomas-Fermi method (1950) corrected for changes in exchange and correlation energies gives a formula from which we have calculated that $q \sim 1.3$.

The phase shifts η_l have been shown by Morse (*loc. cit.*) to depend very little on the precise form of the potential V ; and it is certainly sufficient for our purpose to assume for V a form of type (3) where q is determined by the condition that the screening must be complete, and thus by the equation

$$2\pi^{-1} \sum_l (2l+1) \eta_l(k_M) = Z.$$

For a *positive* charge Z (e.g. a proton) fig. 1 shows that contributions to the displaced charge by $p, d \dots$ components are negligible in the range of momenta occupied by the Fermi gas. For copper for instance, $k_M=0.725$, hence $x_M \simeq 1$ to 1.5. We must therefore have

$$\eta_0(k_M) \simeq \frac{1}{2} \pi Z.$$

The corresponding points in fig. 1 (A for $Z=1$, B for $Z=2$ etc.) are all in a region where at least one bound state exists. Hence a simple Fermi gas cannot provide stable screening round a positive charge through unbound electrons only. This point will be confirmed in § 6, by the study of bound electrons. We must point out however that point A is not very far from the limiting curve corresponding to $\eta_0(0)=\frac{1}{2}\pi$; for actual metals (§ 6) or for finite concentrations of impurities, stable screening by Fermi electrons round impurities with charge $+1$ may be possible.

For *negative* charges Z , no bound states are of course possible and a value of q giving screening by the Fermi electrons is always possible.

We shall now extend these results on screening by Fermi electrons to actual metals. Configurations with bound states will be treated later on by a different method.

§4. SCREENING BY FERMI ELECTRONS. EXTENSION TO ACTUAL METALS

As was shown by several authors (Wannier 1937, Tibbs 1939, Peckar 1946, Mott and Gurney 1948, Slater 1949), the Wigner-Seitz approximation can be applied approximately to the case where an impurity is imbedded in a regular lattice. Thus in a monovalent metal with an impurity atom the conduction electrons may be considered as a Fermi gas of electrons with a certain effective mass (the same as in the pure metal), scattered by the perturbing potential introduced by the impurity, each wave function being modulated by the wave function ϕ_1 , corresponding to the bottom of the band in the pure metal.*

But these results are only approximate. For an electron of momentum k , there is a corrective potential in the Schrödinger equation, of the form $(\nabla\phi_1 \cdot \nabla\phi_2)(\phi_1\phi_2)^{-1}$, if $\phi_2 \exp(i\mathbf{k} \cdot \mathbf{r})$ is the corresponding wave function in the perturbing potential only. This is small for the screened Coulomb fields acting on the conduction electrons; but it is too large for unscreened fields acting on the bound states to give much more than a qualitative description.

Thus, owing to the modulation by the function ϕ_1 , there appear below the 4s band of copper for instance and in an attractive potential of increasing strength, first a 1s, then a 2s, a 2p, . . . states in an interstitial position; or a 4s, . . . state in a substitutional one.† The displacement of charge still follows eqn. (2) near the bottom of the conduction band. The general results on screening obtained for a Fermi gas must therefore still be true for monovalent metals, except perhaps in the limiting case $Z=+1$. This will be confirmed in the more complete picture of the bound states which will be obtained in §5 by another method, based on energetic considerations.

§5. IMPURITIES WITH BOUND ELECTRONS IN METALS

We consider here general characteristics of bound electrons round impurities in metals. Simple applications will follow in the next section, and in §7 a more exact study of a simple case (hydrogen in copper) will justify the approximations made.

The method used so far in this paper has been to study the *phase shifts* η_l of the conduction electrons and deduce the number of bound states from the behaviour of η_l at the bottom of the conduction band. For actual metals, due to the periodic field and other corrections, this is difficult and we shall now obtain more reliable results by considering the *energies* of the bound states and comparing them with the range of energies of the conduction band.

* cf. Appendix (2) for a simple proof.

† The next state is a 4p state, not a 5s one as one should expect if the Wigner-Seitz approximation was still valid for those strong perturbing potentials.

We must first define an energy of reference. We take for our zero, as is usual, the energy of an electron at rest outside the metal. Now, in contrast with the Fermi gas studied in §§ 2 and 3, the bottom of the conduction band in actual metals has a large negative energy $-E_0$. E_0 may be deduced from the energy of sublimation E_s , the first potential of ionization E_1 and the Fermi energy $E_F (=0.6 E_M)$ of the metal (Mott and Jones 1936 b) by the equation

$$E_0 \simeq E_s + E_1 + E_F.$$

Table 1 gives values of E_0 calculated from this formula. Values of E_s are taken from Seitz (1940 a) and Kubaschewski (1951), and E_1 is taken from the Handbook of Chemistry and Physics (1947). Corrective terms for exchange and correlation energies may be neglected if E_0 is calculated by this formula, and E_F may be taken as in a Fermi gas. E_0 is about 15 ev for noble metals and varies from 10 to 6 ev for the Ia group (Li to Cs).

Table 1. Energy Limit E_0 of the Conduction Band in Monovalent Metals (in ev)

Metal	E_1	E_s	E_F	E_0
Li	5.363	1.58	2.84	9.9
Na	5.12	1.125	1.90	8.15
K	4.318	0.93	1.235	6.5
Rb	4.16	0.89	1.075	6.15
Cs	3.87	0.815	0.92	5.6
Cu	7.68	3.53	4.25	15.15
Ag	7.54	3.00	3.31	13.85
Au	9.18	3.92	2.23	16.45

The main difference between the bound states of an impurity in insulators and in metals is therefore the following: in insulators the empty conduction band has a small electron affinity so that bound states have small negative energies. The radius of their orbits is large and they overlap a number of neighbouring atoms. The attractive lattice potential acting on them lowers the energy of their series limit down to the bottom of the conduction band, in accord with Wannier and Slater's theories (cf. § 4); on the other hand the polarization of the overlapped neighbouring atoms reduces the ionization energy of the bound states by a factor K^{-2} (K =dielectric constant, cf. Mott and Gurney *loc. cit.*). In metals on the contrary, impurity electrons have energies below $-E_0$, hence their radii are small and in general do not appreciably overlap the neighbouring atoms. The energy of a bound state must therefore be related to the absolute zero of energy, not the bottom of the conduction band. The influence of the lattice may be treated as a small perturbation and the polarization of neighbouring ions neglected (cf. §§ 7 and 11).

In general the charge displaced in the conduction band does not much modify the form of the bound states, as will be confirmed in §7 in the special case of hydrogen in copper. Ascribing arbitrarily the interaction energy between bound and Fermi electrons to the latter, we may therefore consider the bound states of an impurity as those of an electron moving in the field of the impurity only; then consider the conduction electrons as perturbed by the resulting ion or atom. We may thus use for these bound states the same ionization energies E_i as in vacuo. If $E_i \gg E_0$, the bound state will certainly exist. If $E_i < E_0$ it is very likely to be replaced by some sort of screening by Fermi electrons.

We must therefore compare E_0 with the ionization potentials E_i of the various elements.

§6. EXAMPLES OF IMPURITIES WITH BOUND ELECTRONS IN MONOVALENT METALS

We study first in some detail the simplest possible case, an interstitial proton.

6.1. Interstitial Proton

A proton in vacuo may trap one or even two 1s electrons, forming respectively a H atom and a negative H^- ion, the corresponding electronic energies being -13.5 and -0.7 eV (Massey 1950). Hence in the metal three configurations are a priori possible, depending on the value of E_0 for the solvent metal. The limits given are of course only a rough order of magnitude, owing to the approximations made; energies are in eV:

(a) $E_0 > 13.5$: there will then be screening by Fermi electrons, giving an extra charge in the conduction band.

(b) $0.7 < E_0 < 13.5$: there will then be one bound electron, forming with the proton a hydrogen atom. The field of this atom polarizes the conduction band so as to give no total displacement of charge in the band.

(c) $E_0 < 0.7$: two bound electrons. The hydrogen negative ion thus formed repels one electronic charge in the conduction band.

The simple Fermi gas considered in §§ 2 and 3 corresponds to the case $E_0 = 0$. We see that in this case, in accord with our previous conclusions, there should be two bound electrons in a 1s state. The negative exchange energy between these bound electrons and the conduction band may increase the limit 0.7 eV of this case by several electron volts. But for actual monovalent metals the values of E_0 listed in table 1 are much larger than this and therefore configurations (b) or (c) are the only ones possible. Configurations of type (b) are probable in Cs, Rb, K, Na which thus absorb hydrogen, if at all, in an atomic form, at least for small concentrations.* Noble metals form a limiting case between configurations (a) and (b). In § 7 a detailed discussion is given of this problem for the case of copper.

* The ionic hydrides stable at higher concentrations are probably configurations of type (c) stabilized by the Madelung energy.

6.2. Substitutional Impurities

Table 2 gives the successive ionization energies E_i of a number of elements, E_- being the ionization energy of the negative ion. We see from these values and the values shown in table 1 that dissolved atoms in general lose their first electron to the conduction band of the matrix. All other electrons are retained in bound states, except perhaps for the second electron of some di- and tetravalent metals when the matrix is a noble metal. Thus the second ionization energy of Ga is 20.5 ev and the energy of the electron is well below $-E_0$ ($=-15.4$ ev) for Cu. But for Zn, the difference is smaller, and magnetic properties seem to indicate that the ion Zn^{++} is screened by Fermi electrons (cf. § 10).

Some metalloid elements dissolved in metals to the left of them in the periodic table probably take from the matrix one or more electrons to fill up their np shell, these electrons being in bound states. We have thus a configuration of the type (c) (H^-) studied above. The range of the primary solution is however very limited. At higher concentrations the diamond structure with homopolar binding (type Na_3P) appearing in compounds of alkali metals with P, As, Sb and Bi may be considered as a configuration of this type, where the valency electrons of the metallic atoms just fill the zone derived from the s and p electrons of the metalloid element (Brauer and Zintl 1937). The same is true for ionic crystals (NaCl). In both cases, the configuration of type (c) is stabilized by the Madelung energy.

Table 2. Some Examples of Ionization Potentials E_i . E_- is for Negative Ions

	Be	Mg	Zn	Cd	Al	Ga	Si	Sn	N	As	O	Cl
E_1	(0.2)	(-0.9)			(0.2)		(1.7)		(1.0)		2.3	3.8
E_2	9.28	7.61	9.36	8.96	5.96	5.97	8.12	7.30	14.48	10	13.55	12.95
E_3	18.12	14.96	17.86	16.84	18.74	20.43	16.27	14.5	29.47	20.1	34.93	23.67

§7. SOLUTION OF HYDROGEN IN COPPER

The purpose of this section is to study whether the charge of a proton dissolved in copper is screened by a displacement of the Fermi electrons, or by an electron in a bound state. A self consistent solution is computed for the bound screening in the case of a Fermi gas. Corrective terms are then added to the energy of solution $-E_s$, which is compared with experiment and with the maximum value of E_s one would expect for screening by Fermi electrons.

Bound screening gives a value for E_s in satisfactory agreement with experiment. But Fermi screening perhaps occurs, for the corresponding maximum value for E_s seems a little larger than the experimental value. We are clearly in a limiting case. Results obtained for bound screening are however certainly valid for other phenomena, e.g. round a proton or a divalent substitutional impurity in alkali metals and in x-ray phenomena (cf. §§ 8, 10, 11).

7.1 Proton with a Bound Electron in a Simple Fermi Gas

The proton and the bound electron form a hydrogen atom which polarizes the gas. The process of successive approximations was therefore started with a normal hydrogen atom and plane waves for the gas. Finally we obtained the wave function of the bound electron in the field of the proton and of the charge displaced in the Fermi gas; and the wave function of the Fermi electrons in the field of the proton, of the bound electron and of the displaced charge. The solution was judged sufficiently self consistent when the total charge displaced in the Fermi gas was reduced to 0.1 electronic charge.

We had to compute a number of rapidly oscillating wave functions with s symmetry, for which the usual methods are not very appropriate. We devised therefore the following method.

Decomposition of Schrödinger's radial equation. The Schrödinger equation

$$(d^2\psi/dr^2) + \{K(r)\}^2\psi = 0 \quad (4)$$

may be split into two differential equations of the first order. By analogy with the case where K is constant, we write

$$\psi = a(r) \sin u(r), \quad \text{with} \quad (1/\psi) (\partial\psi/\partial r) = K \cot u.$$

It is easily shown that (4) is then equivalent to the system

$$\frac{\partial u}{\partial r} = \frac{1}{2K} \frac{\partial K}{\partial r} \sin 2u + K : \frac{1}{a} \frac{\partial a}{\partial r} = -\frac{1}{2K} \frac{\partial K}{\partial r} (\cos 2u + 1) \quad (5)$$

and that for an electron with momentum k one has

$$\frac{\partial u}{\partial k} = \frac{k}{2K} \sin 2u + \frac{2k}{a^2 K} \int_0^r \psi^2 dr. \quad (6)$$

For an s wave function, K^2 is of the order of r^{-1} at the origin, hence $u \sim Kr$. The eqn. (5) in u may thus be solved by successive approximations, starting with $u = Kr$. This gives u at infinity by a process in general rapidly convergent. The amplitude a is then obtained by a simple integration.

For large r and if the potential $V = \frac{1}{2}(K^2 - k^2)$ tends to zero rapidly enough at infinity,

$$u \sim kr + \eta$$

where η is the phase shift. A development of a for large r shows that the conditions are: $r^n V \rightarrow 0$ for $r \rightarrow \infty$, with $n=2$ if $k=0$ and $n=1$ if $k \neq 0$; and $\eta = \frac{1}{2}\lambda\pi$ if $k=0$ (λ integer).

It is worth noting that eqn. (6) leads, for large r , to eqn. (2) when we replace u by $kr + \eta$, a by 1 and K by k . For $k \neq 0$ on the other hand, system (5) tends for slowly varying potentials to the *WKB* approximation.

We shall now review the main points of interest concerning exchange, phase shifts and the energy of solution.

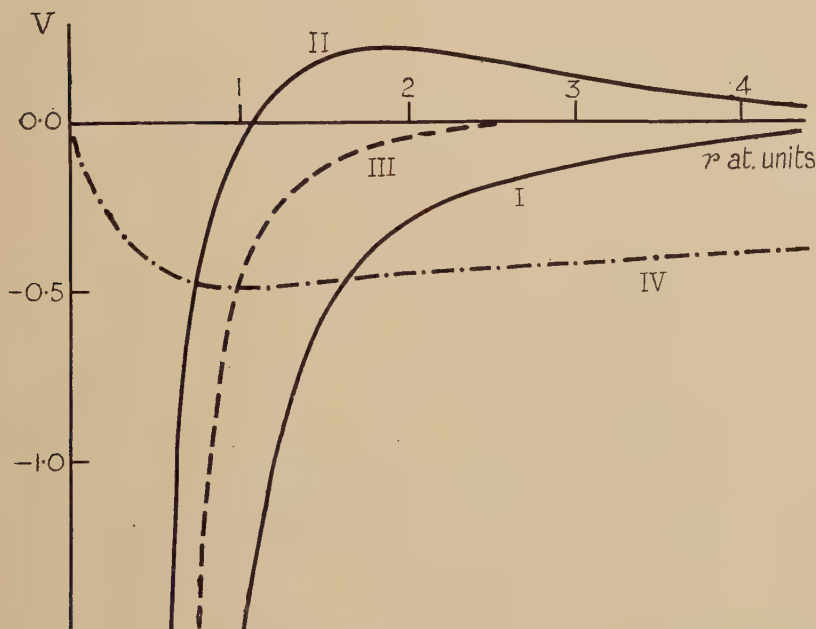
Exchange. The main complication arises from the exchange interaction with the bound electron which introduces a distinction between the spins of the Fermi electrons. Fig. 3 gives the potentials acting on

them. The dotted line (curve III) represents the Coulomb potential provided by the proton and the bound electron. The polarization of the Fermi electrons gives an additional barrier; the potential II, acting on the Fermi electrons with spins anti-parallel to that of the bound electron, becomes therefore slightly negative at large distances. This barrier is masked by the attraction due to exchange in potential I acting on electrons with parallel spins.

The average exchange term ΔV_l due to the bound electron was computed using Slater's approximate formula (1951)

$$\Delta V = -\frac{1}{r} \left[Z_p + 3 \left(\frac{3}{32\pi^2} r^2 \frac{d^2 Z_p}{dr^2} \right)^{1/3} \right], \quad (7)$$

Fig. 3



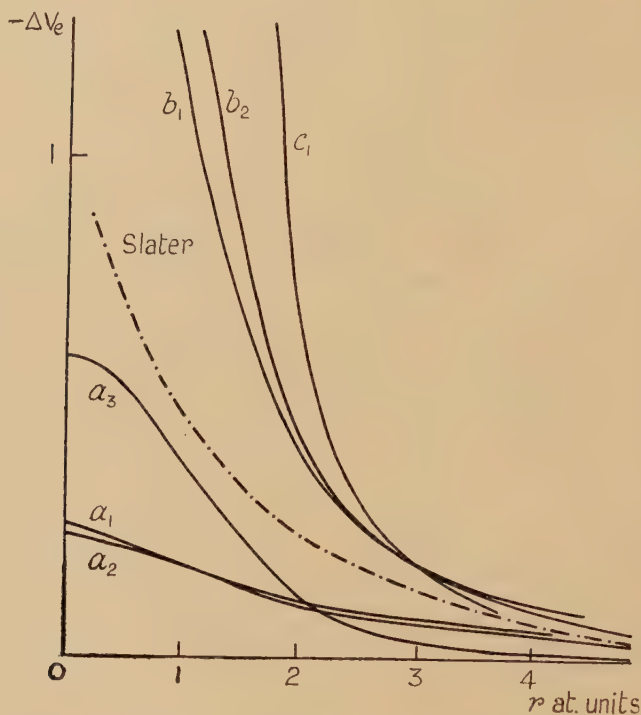
Potentials acting on Fermi electrons with spin parallel (I) and antiparallel (II) to that of the bound electron. The punctuated curve III gives the Coulomb potential of a hydrogen atom. Curve IV gives the complementary potential acting on the bound electron in Slater's method.

where Z_p is the 'effective charge' in Hartree's sense. Correlation between antiparallel spins and changes in exchange interaction between Fermi electrons were neglected in this computation, which does not claim to be very precise.

ΔV_l is somewhat different from the exchange potentials one can calculate for various k and l in the last stage of the computations (fig. 4). Owing to the approximations made on other terms, formula (7) gives nevertheless a satisfactory average value. It has the advantage of giving

for the Fermi electrons of parallel spins a potential I which only differs by a nearly constant term from the potential acting on the bound state (fig. 3, curve IV). The bound state has therefore almost the same form as if it were moving in potential I, as suggested by Slater (1951). Its wave function is practically orthogonal to those of the Fermi electrons with parallel spins, and this simplifies the computation of exchange energies.

Fig. 4



Exchange potentials obtained in the last stage of computations for various l and k , compared with Slater's average value.

a_1 : $l=0$, $k=0.725$; a_2 : $l=0$, $k=0.5$; a_3 : $l=0$, $k=0.3$; b_1 : $l=1$, $k=0.725$,
 b_2 : $l=1$, $k=0.5$; c_1 : $l=2$, $k=0.725$.

Phase Shifts. Fig. 5 and table 3 give the phase shifts and the displaced charges $(2l+1)\pi^{-1}\eta_l(k_M)$ for electrons of both parallel (I) and antiparallel (II) spins in the Fermi gas. Potential I is purely attractive; and all the spherical components with parallel spin are attracted except the s component; this last is repelled owing to the presence of the bound $1s$ electron (cf. § 2). Owing to the mixed character of potential II, attractive at short distances only, the s component with antiparallel spin is slightly attracted at the top of the conduction band, but the p , d , . . .

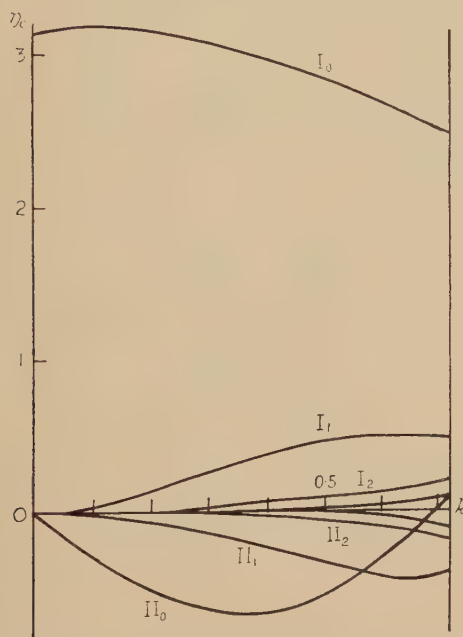
components repelled. The total displaced charge gives rise to the small Coulomb potential barrier described above.

Fig. 5 shows that in the part with parallel spin

$$\eta_0(0)=\pi.$$

One state, corresponding to the bound electron, is subtracted from the bottom of the Fermi gas. This still holds, as a computation showed, when the average potential ΔV_l is replaced by the more exact value obtained for the s component at the last stage of the computations. This point will be used in the study of Hume-Rothery's rules (§ 8). In table 3,

Fig. 5



Phase shifts $\eta_l(k)$ at infinity for electrons with spin parallel (I) and anti-parallel (II) to that of the bound state.

Table 3. Charge Displaced in Spherical Component l for Electrons of Spins Parallel (I) and Antiparallel (II) to that of the Bound Electron. The Bound Electron gives a Charge 1 counted separately

l	0	1	2	3	4	5	6	total.
I	1-0.241	0.450	0.340	0.238	0.170	0.086	0.027	1+1.07
II	0.022	-0.384	-0.355	-0.208	-0.095	-0.034	-0.016	-1.07
I+II	1-0.219	0.066	-0.015	0.030	0.075	0.052	0.011	1+0

the displaced charges have been slightly corrected so as to give no extra charge in the Fermi gas. We see that there is about one additional electronic charge of parallel spin and a corresponding missing charge of antiparallel spin.

Energy of Solution. For the energy of solution of a hydrogen atom, we computed changes in Coulomb and exchange energies directly, by the usual methods (cf. Fock 1933). We thus have :

(a) The change E in Coulomb interaction between proton and bound electron.

(b) The change E' in Coulomb energy for the Fermi electrons. They are in the field of the proton, the bound electron and the charge displaced in the Fermi gas. This gives a Coulomb energy E_1' from which one must deduce the self Coulomb energy E_2' of the displaced charge, counted twice in E_1' .

(c) Finally the exchange energy between bound electron and Fermi electrons of parallel spins, computed for s, p, d components (E_0'' , E_1'' , E_2'') and evaluated roughly for the rest (E_3'').

We find, in atomic units*

$$\begin{array}{rcl}
 E & = & 0.135 \\
 E_1' & = & 0.068 \\
 -E_2' & = & -0.093_5 \\
 \hline
 E' & = & -0.025_5 \\
 E_0'' & = & -0.025 \\
 E_1'' & = & -0.080 \\
 E_2'' & = & -0.030 \\
 E_3'' & = & -0.015 \\
 \hline
 E'' & = & -0.140
 \end{array}$$

Hence for the energy of solution per hydrogen atom we have

$$E_1 = E + E' + E'' = -0.030 = -0.8_2 \text{ ev.}$$

E_1 is therefore *small* and mainly due to the difference between the exchange energy E'' and the change in Coulomb energy E of the bound state. The change in exchange energy between Fermi electrons has been neglected here. It is certainly small. For, by analogy with the results obtained for a vacancy in copper (Huntington *loc. cit.*), it is the difference of two terms of the order of 0.03 atomic units corresponding to the charges attracted and repelled of respectively parallel and antiparallel spins.

7.2 Energy of Solution. Corrective Terms due to the Copper Ions

When hydrogen is dissolved in copper, and not in a simple Fermi gas, several other terms must be considered.

* 1 Atomic unit = 2 Rydberg units = 27.06 ev.

We have first to consider the corrective potential $(\Delta\phi_1 \cdot \Delta\phi_2)(\phi_1 \phi_2)^{-1}$ in which ϕ_1 is the wave function at the bottom of the conduction band. It is constant over a large region round an interstitial hydrogen atom (cf. Fuchs 1935), and the corrective potential may therefore be neglected. Also the electronic density is nearly constant throughout the metal. The approximation of a simple Fermi gas seems therefore satisfactory for the Fermi electrons.*

The bound electron however interacts in two ways with the copper ions. Owing to the exclusion principle, it cannot enter the regions occupied by the $1s^2$ shells of the copper ions. These regions are practically restricted to small spheres and scatter strongly the wave function of the bound state. When the proton is at the common vertex of six atomic polyhedra, it seems probable that the $1s^2$ shells are sufficiently far from the proton and close enough together practically to stop the bound electron and thus to enclose it in a spherical volume of radius $\simeq 3.2$ atomic units. A calculation by Sommerfeld and Walker (1938) enables us to estimate that the energy of the bound state is thus raised by

$$E_2 = 0.05 \text{ Atomic units}$$

without appreciable change in the electronic densities.

On the other hand, the attractive potential wells of the copper ions Cu^+ are neutralized by the charge of the conduction band at the surface of the atomic polyhedra only. Assuming that the bound electron is enclosed in the sphere described above but is otherwise unperturbed we compute that its potential energy is lowered by

$$E_3 = -0.095 \text{ Atomic units.}$$

7.3 Energy of Solution. Comparison with Experiment and with the Energy for Screening by Fermi Electrons

The energy of solution E_s for screening by a bound electron is obtained by summing up the various changes in energy

$$E_s = E_1 + E_2 + E_3 = -0.07_5 \text{ Atomic units} = -2.0_5 \text{ ev.}$$

The experimental value is deduced from measurements of the solubility of H_2 and from the dissociation energy of the hydrogen molecule (Fowler and Smithells 1937). It is

$$E'_s = -0.060 \text{ Atomic units} = -1.6_5 \text{ ev.}$$

Owing to the approximations made, the agreement may be considered as satisfactory.

Without computing in detail the self-consistent solution for screening by Fermi electrons, we may compare with E_s the minimum energy of a configuration of this type: i.e. when the screening charge is just at the bottom of the conduction band (cf. § 5). The Coulomb terms are

$$E_H - E_0 - E_c.$$

Here $E_0 = 15.4$ ev is the energy of the bottom of the conduction band (cf. table 1); $E_H = 13.5$ ev is the electron affinity of the proton, and E_c is

* An effective mass $m^* \neq 1$ should perhaps be used (cf. Mott and Jones 1936 c).

the self energy of the displaced charge. To evaluate this small term, we assume that the displaced charge has a density $\rho(r)$ such as to give Mott's approximate screened coulomb potential (formula 3) :

$$V = r^{-1} \int_0^r \rho dr + \int_r^\infty \rho r^{-1} dr = Zr^{-1} [1 - \exp(-qr)].$$

This gives a density ρ very similar to that of a hydrogen atom. And we obtain

$$E_c = \int_0^\infty \rho V dr = 0.13 \text{ ev.}$$

Hence the total of the Coulomb terms is

$$-2.03 \text{ ev.}^*$$

The exchange term E'' must be of the same order of magnitude as for a vacancy, but with opposite sign (Huntington *loc. cit.*). Hence using Huntington's results

$$E'' \simeq -0.03 \text{ Atomic units} = -0.81 \text{ ev.}$$

The corresponding energy of solution (the sum of E'' and the Coulomb terms) is

$$E''_s = -2.85 \text{ ev.}$$

Owing to the approximations in the calculation, the exact value of this term is devoid of significance. It merely shows that a screening by Fermi electrons is more probable than by bound electrons, the additional charge being near the bottom of the conduction band. Conversely, in a screening by bound electron, if it exists, the electron must be very close below the band. Hydrogen in copper is clearly a limiting case.

§8. HUME-ROTHERY'S RULE

We have now to reconcile our description of valency electrons of impurities in bound states with Hume-Rothery and Jones's point of view (Jones 1934), according to which all the valency electrons are contained in the conduction band. We show how this can be done and describe a mechanism which may explain some experimental deviations from the rules.

In screening by Fermi electrons the impurity merely displaces states in the zone of the conduction band, without creating new ones (cf. § 2).† One more state is occupied for every attracted electronic charge, in accord with the rule.

If a proton keeps a bound electron, it subtracts one state from the bottom of the conduction band (cf. § 7). Similarly a He atom subtracts a state of two electrons (§ 3, fig. 1, point B). Thus every bound electron

* The width of the Fermi band is constant (§ 2); its variation gives no contribution to the energy of solution.

† This was proved for a Fermi gas. Wannier's approximate theorem (§ 3) allows an extension to a general zone structure. No direct general proof has been given as yet.

corresponds to an electronic state subtracted from the band and hence may be said to belong to the band. For this description to be rigorous, the bound and Fermi states should be *orthogonal*. We saw however in § 7 that owing to exchange this condition is almost satisfied for hydrogen in the Hartree-Fock description.

These results may be directly extended to substitutional impurities of the same row as the matrix. Thus Zn, if when dissolved in copper it has one 4s bound electron, probably subtracts one electron from the 4s conduction band of the matrix; Ga, two, etc. As the valency of the impurity increases (Ge, As...), the exchange energy of the Fermi electrons with the smaller s shell decreases, and it is not *a priori* certain that the states subtracted are not fewer than the additional valency electrons. Examples of diminution of valency (Elektronen-rücktritt, Umladung) are in fact cited by Schubert (1948). Thus for example Sb in Cu forms at high temperature a body-centred phase around the Cu_3Sb composition, which is readily explained as a Hume-Rothery β phase if Sb contributes three valency electrons only to the conduction band (Hofmann 1941).

Let us now consider two elements of different rows in the periodic table, Ag in Cu for instance. Ag has a full bound L shell ($4s^2 4p^6 4d^{10}$) which certainly corresponds to states subtracted from the conduction band of Cu. In other terms the perturbing potential introduced by Ag is attractive enough to give phases shifts η_0, η_1, η_2 tending to π at the bottom of the conduction band (cf. § 2). But the same also applies with M states at the *top* of the same zone, where the same perturbing potential acts on 'positive holes' instead of electrons, the corrective potential being also slightly different (cf. § 4). The zone corresponding to the conduction band, therefore, gets rid of the L states at its bottom and accepts M states instead at its top. Ag may be said to modulate the 4s band into a 5s one round itself without altering the number of its states. Similarly, Cd, In, Sn . . . act as Zn, Ga, Ge . . . in Cu, but for the bound L shell introduced below the conduction band without changing the number of states.

Summarizing, the valency electrons of the impurities are in general in the conduction band or in bound states subtracted from the band; they are the only ones to have this property. They may indeed be considered as a mere extension of the band.

We may note in conclusion that the configuration corresponding to a negative ion H^- does not fit with this description. In the Hartree-Fock approximation, the conduction electrons move in a repulsive potential which does not subtract any state from the bottom of the conduction band (cf. § 6.1). This approximation is probably satisfactory. For the s part of the conduction band is repelled at the bottom of the band and is replaced by the p component at the top, owing to overlap of the s and p bands (Krutter 1935). The 1s bound state is therefore nearly orthogonal to it and mainly subtracted not from the conduction band but from the rest of the system of s bands. Such a configuration requires the formation

of a repulsive ion, i.e. at least a negative interstitial ion or a neutral substitutional atom in monovalent metals, or a monovalent substitutional ion in divalent metal, etc.

This behaviour seems to occur mainly when the components of the alloy have very different valencies. Thus Schubert gives numerous examples for alloys of polyvalent metals with transitional and rare earth elements (ZnNi, pure Ca, MgLa, etc.). In a similar way, Raynor (1949) assumes that transitional elements subtract conduction electrons from aluminium to build up negative ions with complete d shells. Truly *ionic* configurations may be considered as an extreme case, where the concentration of negative metalloid ions is large enough to empty completely the conduction band of the metal.

The difference in magnetic properties of Ni, Pd and Pt dissolved in noble metals may be explained in the same way. In Hume-Rothery alloys, it is well known that the transitional elements normally make no contribution of electrons to the conduction band, hence dissolve as neutral atoms. One may ask what sort of neutral atom. Ni for instance may dissolve as $3d^{10}$, $3d^9 4s$ or $3d^8 4s^2$, etc., the 4s states remaining bound. As already explained for H^- in Cu, the possible 4s bound states are *not* to be subtracted from the conduction band. It is thus probable that, when dissolved in small concentration, these transitional elements have the same configurations as in a gas, paramagnetic $3d^8 4s^2$ and $5d^8 6s^2$ for Ni and Pt respectively, diamagnetic $4d^{10}$ for Pd. Pd is actually diamagnetic in Cu or Au, Ni and Pt rather strongly paramagnetic (cf. for instance Vogt 1951).

§9. APPROXIMATE EVALUATION OF ENERGIES OF SOLUTION IN MONOVALENT METALS

We devote the last sections of this paper to applications to energies of solution (§ 9), paramagnetism of alloys (§ 10) and x-ray spectra (§ 11).

The assumption of bound screening, necessary for As or Al in Cu, probably gives reasonable results for the energy of solution even when screening by Fermi electrons actually takes place, as is probable for Zn in Cu. In calculating these energies we shall take as an example aluminium in copper.

The energy of solution of Al in Cu is the change of energy of an aluminium atom taken from the metal and put in substitution into a copper lattice. We obtain it by the following cycle :

(1) First, take an aluminium atom from the bulk metal. If H_s is the sublimation energy, the work required is H_s .

(2) Then take one electron from it, leaving an Al^+ ion. The work done is the ionization energy E_1 .

(3) Take a copper ion out of the matrix, putting an aluminium ion instead. The work done will be small; we denote it by δ .

(4) Put the electron on the copper ion. If E_1' is the ionization energy of Cu, the work done is $-E_1'$.

(5) Put the copper atom back in the lattice; the work done is $-H'$ where H'_s is the sublimation energy of Copper.

The total work done (the energy of solution E_s) is

$$E_s = H_s + E_1 - H'_s - E'_1 + \delta.$$

Table 4 gives the computed values of E_s for Cu and Ag alloys, compared with experiment. The measurements are accurate for **Cu Zn**, **Ag Zn** and **Ag Cd*** only (Birchenall 1949). The other experimental values were obtained by fitting some reasonable hyperbolic curve with results obtained at rather high concentrations (cf. Wagner 1940, Smithells 1949). Values for H_s are taken from Seitz (1940 a) and Kubaschewski (1951).

Discrepancies between computed and experimental values must be ascribed to the term δ which we neglected altogether. δ contains several contributions. First, a very small positive elastic energy, arising from the difference in atomic radii between solvent and solute elements. Then a term corresponding to Coulomb and exchange interaction between the

Table 4. Energies of Solution in Cu and Ag (in ev)

	H'_s	E'_1	H_s	E_1	$+E_s$ comp.	$+E_s$ exp.
Cu Zn	3.53 ± 0.09	7.68	1.365 ± 0.02	9.36	-0.485 ± 0.11	-0.35_5
Cu Al			3.30 ± 0.22	5.96	-1.95 ± 0.31	(-1.5)
Cu Sn			(3.38)	7.3	(-0.53)	(-0.5)
Cu As			(1.31)	10	(+0.10)	(-2.7)
Ag Zn	3.00 ± 0.06	7.54	1.365 ± 0.02	9.36	$+0.165 \pm 0.08$	-0.18
Ag Cd			1.16 ± 0.02	8.96	-0.42 ± 0.08	-0.19

conduction band and the bound states of the impurity. This is small for H in Cu (§ 7), but difficult to evaluate for substitutional impurities in noble metals (an evaluation will be made for light metals in § 11). Finally, the change in Van der Waals energy between ion cores is probably not negligible.

We must thus not expect any great accuracy. Except for **Cu As** and **Ag Zn** the agreement is actually reasonably good. It is interesting that bound screening gives satisfactory results even for atoms with an even number of electrons (Zn, Sn) for which two electrons are more probably in the conduction band. We may also point out that to put the three valency electrons of Al in the conduction band of Cu would require at least +15 ev, as is shown by a simple computation, in contrast with the small negative energy of solution of this element. It is therefore impossible for elements like Al, Sn, As . . . to have all their valency electrons in the conduction band of the matrix.

* **Cu Zn** means Zn dissolved in Cu.

§10. MAGNETISM OF ALLOYS WITH NOBLE METALS

Measurements of the susceptibility of alloys would probably provide the most direct check of some of our results, and especially on the existence of incompletely filled bound states. We analyse in this section the few experimental data available.

We shall not deal here with magnetism of alloys of noble metals with transitional elements, the interpretation of which seems to be of a certain complexity (Vogt 1951). We refer only to a qualitative explanation proposed in § 8 for para- and diamagnetism in alloys of Ni, Pd, Pt, with Cu and Au.

From the few measurements available on alloys with polyvalent elements, the only reliable ones seem to be those on **Cu Zn** (Endo 1925) and **Cu Al** (Vogt and Harms 1943).

For **Cu Al**, we assume that the contribution of the Fermi electrons of Al is the same as in pure Cu, and that the variation of atomic susceptibility is due to the replacement of Cu^+ by Al^+ ions. The same assumption would lead, for Cu Au for instance, to a linear variation of χ which is actually followed within 1 to 2 units $\times 10^6$ (Shimizu 1932).

Let then

$$\chi = x\chi_{\text{Al}} + (1-x)\chi_{\text{Cu}}$$

be the susceptibility for atomic percentage 100*x* of aluminium. χ_{Cu} is the susceptibility of pure copper, and χ_{Al} is the contribution of the aluminium atoms, expressed in CGS units. For every aluminium atom, contributions to χ_{Al} are from the Al^{+++} ion, the $3s^2$ shell and the Fermi electron. The unknown susceptibility of the two $3s$ electrons of Al^+ was computed by Slater's method (1930) using for the screening constant the value deduced from the ionization potential of Al. This gives $\chi_1 = -21 \times 10^{-6}$. Taking for Al^{+++} : $\chi_2 = -2.5 \times 10^{-6}$ (Pauling 1927); for Cu^+ : $\chi_3 = -14.7 \times 10^{-6}$ (Hartree and Hartree 1936); and for Cu metal: $\chi_4 = -5.3 \times 10^{-6}$ (Vogt and Harms *loc. cit.*), we have for Al in Cu

$$\chi_{\text{Al}} = \chi_1 + \chi_2 - \chi_3 + \chi_4 = -14.1 \times 10^{-6}.$$

The experimental value is

$$\chi'_{\text{Al}} = -12.2 \times 10^{-6}.$$

Owing to the approximations made, the agreement is satisfactory.

For **Cu Zn**, it is interesting to note that the *diamagnetism* is increased when Zn is added. If the additional $4s$ electron of Zn was in a half occupied bound state, one would have expected a strong paramagnetic term to appear. We must therefore assume a screening by Fermi electrons (cf. § 6). Qualitatively, the effective mass in the Fermi gas increases slightly when Zn is added, so as to accommodate the extra electrons in the band whose width is unchanged. Hence the paramagnetic term decreases and the Landau diamagnetism increases (Mott and Jones 1936 d). Quantitatively however, this effective mass varies with the energy in the conduction band: and it is simpler to assume as for the energies of solution

(§ 9) that the supplementary electronic charge has nearly the same distribution in space as a 4s bound electron in the field of Zn^{++} . To compute the diamagnetism of this charge, we use the same method as above for the 3s² electrons of Al^+ , and take for the difference between the susceptibilities of Zn and Cu : $+2 \cdot 10^{-6}$ (Pauling *loc. cit.*). This gives

$$\chi_{\text{Zn}} = -24 \cdot 1 \times 10^{-6}.$$

The experimental value is

$$\chi'_{\text{Zn}} = -24 \cdot 5 \times 10^{-6}.$$

The agreement is better than the approximations warrant.

Other measurements (Davis and Keeping 1929, Spencer and John 1927) show rather strong paramagnetic increases of the magnetic susceptibility for **Cu** Mg, **Ag** Pb and **Cu** Sb. They are probably due to paramagnetic properties of Mg^+ , Pb^+ and Sb^{++} ions. The variations with concentration and temperature should however be more thoroughly investigated before any definite conclusion could be made.

One could expect a priori a paramagnetic susceptibility inversely proportional to the temperature, as it is the case for transitional metals like Co, Fe in noble metals (Vogt 1951). Here however, the paramagnetic bound states have much larger radii and a direct coupling is probable already for small concentrations. We would also expect a rather strong indirect coupling by the electrons of the conduction band, which are strongly perturbed (§ 7). These two reasons seem to indicate that the paramagnetism should not vary much with temperature.

§ 11. X-RAYS AND OPTICAL TRANSITIONS IN MONOVALENT METALS AND THEIR ALLOYS

We shall now use the results obtained in the preceding sections to study the absorption and emission of x-rays by monovalent metals.

The absorption of an x-ray quantum by a metal is usually described as a transition of an inner electron to the continuum above the top of the conduction band. This process leaves however a metallic ion with an excess positive charge. Just as for the case of a Zn ion in copper, the conduction electrons of a monovalent metal cannot remain unaltered round this charge, but must provide some sort of screening, the most stable form being given by a bound electron below the conduction band. This electron, together with the ion which has lost an electron from an inner shell, forms an excited monovalent ion which replaces a normal monovalent ion without much effect on the conduction electrons. In calculating the energy of the absorption edge, we may therefore neglect as before (§ 9) the changes of energy in the conduction band, and consider only the excitation of the monovalent ion.

The absorption process thus involves several *simultaneous* electronic transitions, and the selection rules only apply to the total resulting changes of symmetry. The positive hole in an x-ray level and the additional bound electron form together an exciton in Wannier's sense

(Wannier *loc. cit.*). We may thus state that in the main absorption or emission processes the exciton of lowest energy is respectively created or annihilated. We shall see that the presence of the Fermi gas broadens the emission into a band and makes possible transitions to an exciton of the same symmetry as the initial state. In metals, the successive excitons are widely separated in energy; those of energies higher than the fundamental one thus give rise to a fine structure on the short wave length side of the main emission or absorption edge.

We develop the simplest case, Li K, then review various other cases where simple quantitative checks are possible: $\text{NaL}_{\text{II, III}}$ and optical transitions in noble metals; shifting of K absorption edge in Na and NaCl, K and KCl; and finally some K transitions in Cu alloys.

11.1 K Absorption and Emission in Li

We shall describe the transition process in a qualitative way, then check energy results with experiment for both absorption edge and fine structure.

(a) We show first that the energy of the absorption edge corresponds to the excitation of a Li^+ ion from the state $1s^2$ to $1s\ 2s$. During the absorption (fig. 6) an electron is taken from the $1s^2$ shell and sent into the continuum in a p state B. The hole A and the remaining $1s$ electron D normally go out of the $1s^2$ band into what we may call the $1s'$ bound state* of a definite atom (A' , D'). This transition, which takes place when the Li atom is in a gas, reduces the absorption energy by DD' . According to our preceding results the now localized positive charge A' requires screening, the most stable configuration being obtained with a bound electron C' in a $2s'$ state. The field acting on this electron is mainly due to the Li^{++} ion and the energy of the conduction band is not very much affected by the excitation $1s^2 \rightarrow 1s'\ 2s'$ of the Li^+ ion. Small corrections, corresponding to the term δ in § 9, will be considered later.

In conclusion, we picture the K absorption by a triple electronic transition: a $1s$ electron A, going into B in the continuum, leaves a positive hole which goes with the remaining $1s$ electron D into a localized $1s$ state ($A'D'$). The charge of the positive hole is screened by an electron C' produced from the Fermi band and put into the $2s$ state of the excited Li atom. The positive hole A' and the bound electron C' constitute a Wannier exciton (Wannier *loc. cit.*). The total energy for the absorption is approximately

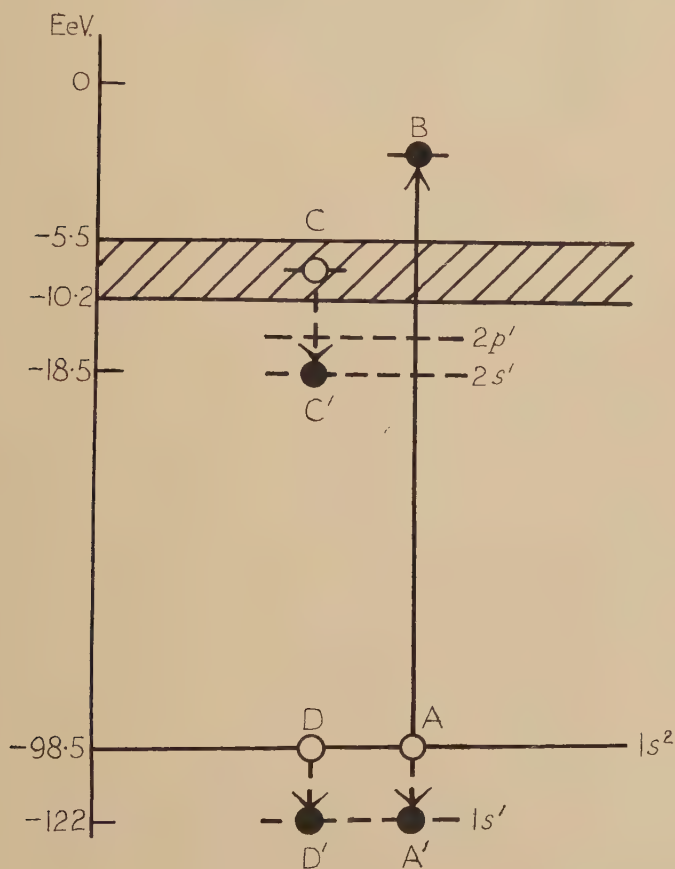
$$AB - CC' - DD'.$$

The auxiliary transitions (C' , DD') are not allowed in a one electron theory. They must be considered as induced by the perturbation of the photon at the same time as the main transition AB : AB , CC' , DD' represent a unique transition of the whole electronic system. Similar processes occur in every polyelectronic system, the simplest being the K absorption of helium where the remaining $1s^2$ electron goes into a $1s'$ state.

* Dashes for states of the excited atom.

(b) It may be shown that the absorption has a non-vanishing probability when the hole C is at the *top* of the Fermi distribution. There is therefore a discontinuous increase of absorption at the absorption edge, when the energy of the incident photon becomes sufficient to raise electron B above the Fermi distribution. This point is in accord with experiment. It may, however, only be proved by a study of *all* the Fermi electrons, and we intend to develop it in a later paper.

Fig. 6



K absorption process in Li.*

We shall only compute here the energy of this *absorption edge*, when B and C are both at the top of the Fermi band (fig. 6). The main term is the excitation energy E_I of $\text{Li}^+ 1s^2 \rightarrow 1s 2s$. This is given by optical data (Bacher and Goudsmit 1932).

We make two corrections. First, an exchange energy $-E_c$ between the $2s'$ electron C' and the $2s$ conduction band. We use here Fock's

* A' should be a white circle (as A).

formula (Fock *loc. cit.*) with Slater's approximate wave functions for both 2s and 2s' (Slater 1930). A straight-forward computation gives for the exchange between ($n_1 l_1$) and ($n_2 l_2$), in atomic units

$$-E_e = A a_1^x a_2^y (a_1 + a_2)^{1-x-y} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

with $x = 2n_1 + 1$, $y = 2n_2 + 1$, $a = 2Z_e n^{-1}$,

where Z_e is the 'effective charge' acting on the electron, computed from Slater's empirical rules. A is a numerical factor given in table 5 for various cases. As we used for 2s a bound wave function non-orthogonal to 2s', values computed by (5) are only approximate.

Table 5. Numerical Values of Factor A

	2s 2s	2s 2p	3s 2s	3s 2p	3s 3s	4s 3s	4s 4s
A	93	120.6	257	332.5	1,056	3,328	13,200

As a second correction, we compute the small negative energy $-E_L$ due to the attraction of 2s' by the potential of the lattice in the neighbouring atomic polyhedra. We assumed as in § 7 that the form of 2s' was not perturbed; the potential of Li^+ was taken from Fock and Petrashen (1935), and the density of Fermi electrons assumed to be uniform.

The presence of a polarized charge in the conduction band alters slightly the form of the 2s' wave function, and hence changes its Coulomb interaction with the Li^{++} ion (cf. § 7). This is however smaller than for H in Cu where the bound electron is nearer to the conduction band. Assuming the same displaced charge as in Cu H, we found a maximum value of 1 ev. We neglected here this term altogether, and owing also to the approximation in the exchange term, we do not claim an accuracy better than 1-2 ev.

Table 6. K Absorption Edge and Fine Structure for Li (Energies in ev)

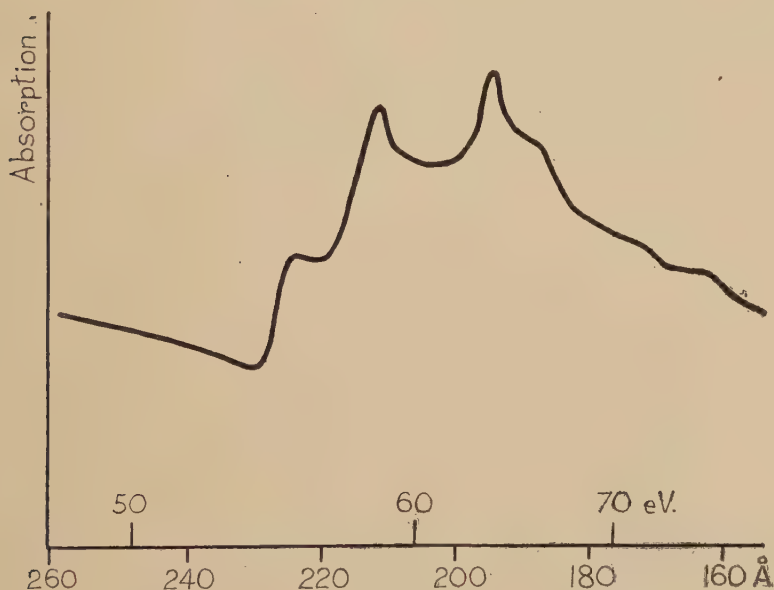
	E_I	E_e	E_L	Total	$h\nu$ abs.	$h\nu$ em.
2 s'	58.7	3.1	0.15	55.4 ₅	55	54.5
2 p'	61	3.7 ₂	0.15	57.1 ₃	58.5	
3 p'	69	3.00	2.15	63.8 ₆	63.5	

The total $E_I - E_e - E_L$ thus computed (table 6) is in satisfactory agreement with the experimental value, taken from Skinner and Johnston (1951). Two points may be noted. First, the energy of the 2s' electron is about -18.5 ev, definitely below the conduction band which extends from -5.5 to -10 ev. Also a screening by two bound electrons (2s²) and a positive hole in the conduction band is a priori possible (analogous

to configuration (c) §6). Using Slater's approximation for the energy of bound states, one finds however that such a configuration is less stable by about 3 ev than the screening by one electron described above. A configuration $1s\ 2s^2$ could therefore play a role only in the fine structure.

(c) The *fine structure* to the short-wave side of the absorption edge is interpreted by Jones and Mott (1937) as a Kronig structure. Assuming that the average density of states is the same as in a free electron gas, reflexions on (110) and (200) planes would correspond to two peaks of absorption at about 3.8 and 12.3 ev from the absorption edge. Two strong lines are actually visible (fig. 7, taken from Skinner and Johnston 1937) at 3.5 and 8.5 ev from the absorption edge.

Fig. 7

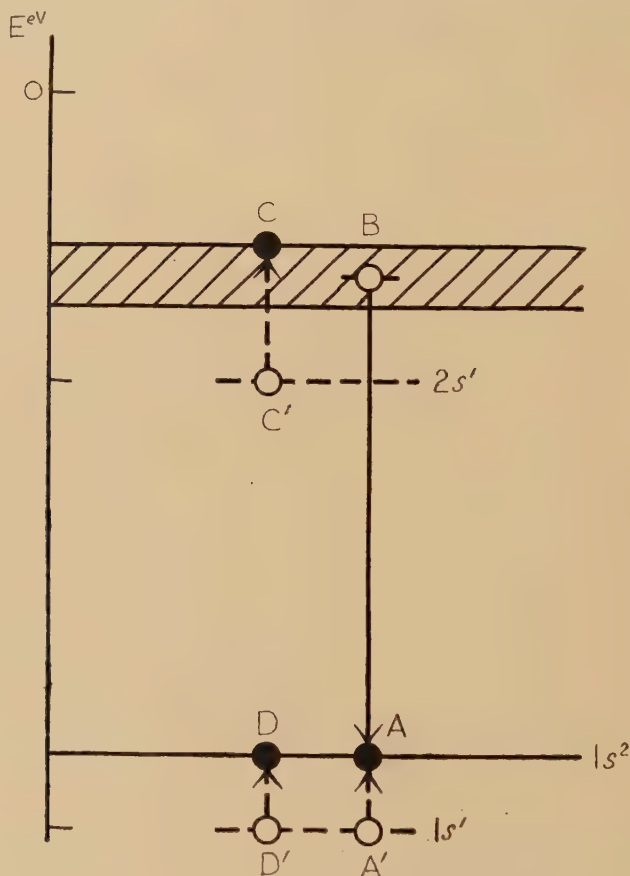


Li K absorption (after Skinner and Johnston).

The preceding analysis suggests however an alternative interpretation. The screening at the end of the absorption process could be obtained by a $2p'$ (or $3p'$. . .) electron instead of $2s'$. These configurations require a higher energy than the fundamental one with $2s'$; but owing to their p symmetry, a direct $1s \rightarrow 2p'$ transition is possible, without affecting the Fermi electrons. Hence we expect the presence of strong absorption lines on the short-wave lengths of the absorption edge. Table 6 gives their position computed by the same method as for the absorption edge. A stronger dependence of the fine structure near the absorption edge upon atomic than upon lattice structure has been reported by several authors (see for instance Coster and Kiestra 1948, Sanner 1943) and may probably be explained in a similar way.

(d) The *emission process* is just the reverse of absorption, starting with a bound electron $2s'$ screening a hole in the $1s$ shell (fig. 8). The edge of the emission band corresponds to the case when B is at the top of the Fermi band. Thus absorption and emission edges must correspond to the same energy, a well known fact in metals (cf. O'Bryan and Skinner 1934).

Fig. 8



K emission in Li.

Summarizing, both absorption and emission edges correspond to a transition which may be represented by an internal $1s^2 \rightarrow 1s\ 2s$ excitation of a Li^+ ion. The fine structure in the absorption may be attributed to further excitations $1s\ 2p$, $1s\ 3p$. . .

A similar quantitative study is possible for other monovalent metals as long as the excitation energy E_I of the monovalent ion may be deduced from some other experiment. We shall now review a few examples where this is the case.

11.2. Na $L_{II, III}$ Emission and Absorption

The description is similar to that for Li K, a positive hole in the 2p level being screened by a bound 3s electron. The small peak at the top of the emission band (Skinner 1940) may be due to the allowed direct transition $2p^6 \rightarrow 2p^5 3s$. Table 7 (first row) gives the computed and observed values for the emission edge. The two values for E_I correspond to the two states $2p^5 3s$ of momentum $J=1$, which give rise to the doublets $L_{II, III}$, $K\alpha_{12}$, $L\eta_i$, etc. . . . E_e and E_L are computed as before (values for Na^+ taken from Fock and Petrashen 1934).

Skinner's emission curve shows a small satellite at 33 ev followed by a more extended one between 42.5 and about 39 ev. The first satellite is readily explained by assuming an initial screening by a 3p instead of a 3s electron. Its intensity is small, for the 3p—2p transition is possible by the intermediary of the conduction band only. The 3d, 4s . . . excited states seem responsible for the second (extended) satellite. The 3s, 3p

Table 7. $L_{II, III}$ Absorption and Emission for Na (energies in ev)

	E_I	E_e	E_L	Total	$h\nu$ abs.	$h\nu$ em.
3 s'	32.77			30.1 ₂	30.5	30.25
	33.15	2.27	0.37	30.5		
3 p'	36.17	2.75	0.37	33.0 ₅	30.7 (32.4)	30.55 33.0
Series Limit	47.0 ₅	0	5.16	41.9		42.5

states are below the conduction band, as is easily deduced from data in table 7. The 3d, 4s . . . states on the contrary are within the conduction band and should be replaced by equivalent screenings by Fermi electrons (cf. § 9). Their series limits correspond to an electron displaced to infinity in the metal, and hence to the top of the Fermi band whose energy is computed with the conventions of § 5 by

$$-E_L = -E_s - E_1 + 0.4 E_M = -5.1_6 \text{ ev.}$$

The second and third rows in table 7 give the corresponding computed and observed energies.

We may note that the series limit ($h\nu=42.5$ ev) corresponds to the L emission of a singly ionized ($1s2s^2 2p^6 3s$) atom, and not to a doubly ionized one as is usually stated for satellites. It corresponds in fact to the mechanism of emission as usually described. But the screening by a 3s electron reduces the energy from 42.5 to 30.2 ev.

The absorption edge, as the values in table 7 due to Rule (1944) show, is in good agreement with the emission edge. To the short wave side of it, one sees in O'Bryan's curves (1940) an absorption peak at c 32.4 ev

which may correspond to the satellite at 33.0 ev. Here too, the Kronig structure would not give very different results; one would expect maxima at 2.55, 8.25 and 13.95 ev from the absorption edge.

In conclusion, we suggest that the mechanism may be analogous to that for Li K. The satellites and secondary absorption peak may be explained by the presence of more excited Na^+ ions.

11.3. Displacement of K Absorption Edge for Na and NaCl, K and KCl

The position of the absorption edge of an element depends upon the nature of the solid in which the process takes place (cf. Pauling 1929 for instance). We give two examples where the major part of the displacement may be computed, and analyse the fine structure of the K edge of Na metal.

(a) In NaCl, the Na K absorption edge (series limit) has an energy $h\nu$ given by (Mott and Gurney 1948 b)

$$h\nu = E - W_L - P - \chi.$$

Here E is the ionization energy of a 1s electron in a Na^+ ion: $-P$ the energy due to the polarization in the crystal round a point charge; $-\chi$ is the energy of the bottom of the band; finally $-W_L$ is the lattice energy per ion pair, mainly due to the electrostatic interaction between ions.

Table 8. Displacement of Absorption Edges of Na and K (ev)

	E_s	E_e	E_L	W_L	P	χ	$h(\nu-\nu')_{\text{comp}}$	$h(\nu-\nu')_{\text{obs}}$
NaCl	14.9 ₆	2.1 ₆	0.2 ₅	10.3 ₃	1.5 ₂	0.5 ₃	5.0	4.0
KCl	11.8 ₂	0.2 ₂	1.1 ₀	8.7 ₈	1.4 ₄	0.0 ₇	2.8 ₅	3.1

In the metal on the other hand, we assume that the point charge is screened by a 3s electron: the sodium ion is thus in the state (1s 2s² 2p⁶ 3s). The frequency $h\nu'$ of the absorption edge is thus given by

$$h\nu' \simeq E - E_s - E_e - E_L,$$

where $-E_s$ is the ionization potential of the screening 3s electron and $-(E_e + E_L)$ the usual corrections for exchange and lattice interactions.

The difference between the two energies, giving the displacement of the absorption edge, is therefore

$$h(\nu-\nu') \simeq (E_s + E_e + E_L) - (W_L + P + \chi).$$

We take as an approximation for the unknown E_s the ionization potential of the 3s electron in Mg^+ , which is in a similar field, that is to say the second ionization potential of Mg. Thus $-E_s = 14.9_6$ ev. The quantities χ , P and W_L are taken from Mott and Gurney (*loc. cit.*). As we consider Na and not Cl, we must however change the sign of the electrostatic term in W_L . E_e and E_L are computed as before. Table 8 gives the computed value for $h(\nu-\nu')$, in satisfactory agreement with experiment (Rule 1944).

In a similar computation for KCl, the potential of K^+ used for E_c is that of Hartree and Hartree (1938) and $-E_s$ is taken as the second ionization potential of Ca. The observed displacement is taken from Lindh (1930).

(b) The *fine structure* on the short wave side of the absorption edge in Na metal may be studied as for Li K. The absorption edge itself is decomposed into three parts (K_1 , K_2 , K_3 in Rule *loc. cit.*), which may correspond to screening by 3s, 3p and 4s electrons. The absorption then increases slowly up to a maximum A which is approximately at the same wavelength as the series limit. Table 9 gives the computed and observed energies. In the first column, the unknown values of E_1 are replaced by the corresponding values for the normal Mg^+ ion. As for Li K, the fine structure could also be explained by a Kronig structure. One would expect a maximum of absorption for the energies listed on the right in table 11; the agreement is of the same order.

Table 9. Fine Structure of Na K Absorption Edge (ev)

	ΔE_I	$-\Delta E_e$	$-\Delta E_L$	Total	$\Delta h\nu$ exp	Deno- mina- tion	Expected Kronig Structure	Deno- mina- tion
3s'					0 ± 0.1	K_1		
3p'	4.4_1	-0.4_3	0	3.9_8	$3.0_5 \pm 0.8$	K_2	2.5_4	(110)
4s'	8.6_3	0.2_3	-1.6_0	7.2_6	5.6 ± 0.1	K_3	8.24	(200)
Series limit	14.9_6	2.1_6	-4.2_1	12.2_1	$10.2_1 \pm 1.4$	A	13.94	(211)

11.4. Optical Absorption in Noble Metals

Copper, silver and gold show absorption in the blue or ultra-violet; to this is due the colour of copper and gold. As was pointed out by Mott (Mott and Jones 1936 e), the main absorption seems due to internal photoelectric effect, with excitation of a d electron. We shall show that the same model as for x-rays can be used, and shall relate the absorption energy to the energy separation between the s and d parts of K and $L_{II, III}$ emission bands.

Optical absorption in noble metals is usually described as a transition of an electron from the d band to the top of the Fermi band, without screening. A simple computation shows that in copper the top of the Fermi band is at about -8.2 ev. The first ionization potential of the d shell, given by the second ionization potential of Cu, is -20.34 ev; and the absorption edge should therefore be at about 12.2 ev, very different from the actual value of about 2.1 ev.

As for x-rays, we must therefore assume a screening electron, 4s for Cu. Thus the energy of the absorption edge should be near to the excitation energy E_1 of the inner d shell ($3d^{10} \rightarrow 3d^9 4s$ for Cu). Table 10 gives the

value of E_I for Cu, Ag and Au (Cu and Ag : Bacher and Goudsmit *loc. cit.*, Au : Platt and Sawyer 1941). The correction $-(E_c + E_L)$ is probably small, as for **Cu** H (§ 7) ; but Slater's approximation is very poor for such heavy atoms and a more exact computation would be necessary. The parallelism between E_I and the experimental values taken from Mott and Jones (1936 e) is however satisfactory.

As shown by Rule (*loc. cit.*), the $L_{II,III}$ absorption in elements Na to Cl is equivalent to the difference between a $K\alpha_{1,2}$ and the normal $K\beta_x$ emissions, and its energy is thus equal to the $K\alpha_{1,2} - K\beta_x$ separation. In a similar way, the energy of the optical transition in copper must be equal to the separation between the s band edge and the secondary d peak in the Cu K and $L_{II,III}$ emissions. For Cu K, the distance between $K\beta_2(4sp \rightarrow 1s)$ and $K\beta_5(3d \rightarrow 1s)$ is about 3 ev (Bearden and Friedman 1940). For Cu $L_{II,III}$ recent measurements by Cauchois (1951) give about 2.5 to 3 ev. Both values agree with the peak 2.5 ev of the optical absorption.

Table 10. Optical Absorption in Noble Metals (ev)

		E_I	$h\nu$ exp limit	$h\nu$ exp peak
Cu	4s	2.90	2.1	2.5
Ag	5s	5.40	4.0	4.4
Au	6s	3.42	2.5	

11.5. Al K Emission Band in **Cu** Al and similar Emissions in other Alloys

We shall now extend our results to alloys of noble metals. We start with **Cu** Al as an example, and verify some of our conclusions on other alloys.

Recent measurements by Cauchois (1950, 1951), completing previous ones by Yoshida (1935) and Farineau (1939), present some characteristic features: the $K\beta$ emission band is divided between a rather sharp peak at the edge (width about 2 ev) followed by a second peak (width about 4 ev) at 4.5 to 5 ev from the edge ; the position of the edge is practically independent of Al concentration.

We picture as in § 9 an Al atom dissolved in Cu as an Al^+ ion plus one Fermi electron having the same energy limits and density of states as in pure copper, but with an s component repelled and p, d, . . . components attracted round the Al atoms.

Before the K emission of an aluminium atom the positive hole in the 1s shell of Al is therefore screened by an additional 3p electron, so that the configuration is $(1s\ 2s^2\ 2p^6\ 3s^2\ 3p)$, similar to an excited free ion. The electron which fills the positive hole during emission may come from the conduction band, the 3p state or the $3s^2$ shell. The three cases are described in fig. 9 (a), (b), (c). In (a) the additional 3p electron must disappear during emission ; and in (c) the transition is allowed only by the intermediary of the conduction band.

Transition (a) gives what we may call the weak 'normal' emission band, similar in form to $K\beta_2$ of Cu. Superimposed at the emission edge, there is a strong peak corresponding to the direct transition (b). Finally, displaced towards the long wavelengths by an amount equal to the excitation energy of $3s^2 \rightarrow 3s3p$ in Al^+ , there is a second and stronger emission band, also similar in form to Cu K. Table 11 (first row) gives the computed value for this displacement, in good agreement with Cauchois and Farineau's results. If now the Al concentration increases, the form and position of the conduction band may vary. But this has very little effect on the energy of excitation of the Al^+ ion, at least as long as the $3s^2$ band is not too much widened. Thus the absolute position of the emission edge remains practically constant.

Fig. 9

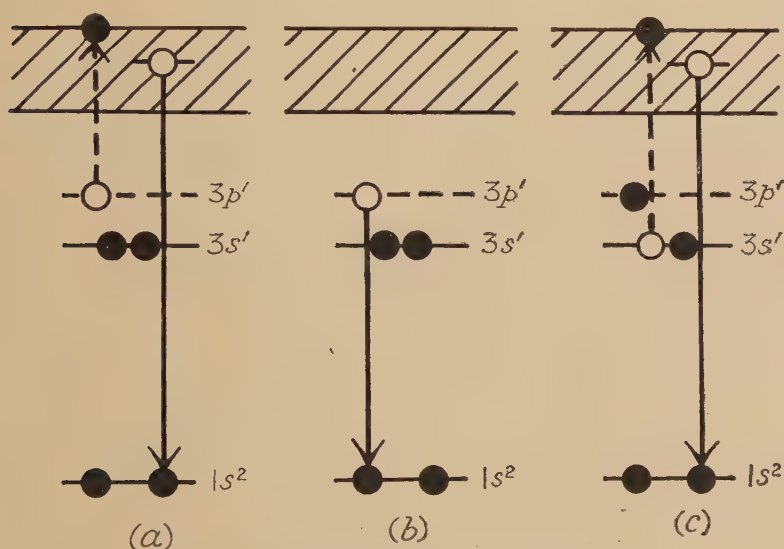
Three types of AlK emission in **Cu Al**.

Table 11. Separation between Secondary Peak and K Emission Edge (ev)

	ΔE_I	$-\Delta E_e$	$-\Delta E_L$	Total	$\Delta h\nu$ exp
Cu Al 3s 3p	-4.6 ₂	+0.0 ₁	0	-4.6 ₁	-4.5 to -5
Cu Zn 4p	-5.9 ₈	+0.6 ₁	0	-5.3 ₇	c. -5

In conclusion, we have satisfactorily explained the various characteristics of Al K emission in **Cu Al**. Some of the results are valid for other alloys. Thus :

(1) The position of the emission edge is practically independent of the (not too large) concentration of solute atom.

(2) K emission band of polyvalent elements dissolved in Cu presents one or more secondary peaks on the long wavelengths side (e.g. Bearden and Friedman *loc. cit.* for **Cu** Zn; Skinner and Johnston 1938 for **Cu** Be). These peaks may be interpreted as inner transitions producing excited solute atoms. Table 11 (second row) gives the computed displacement for **Cu** Zn corresponding to an excitation $4s \rightarrow 4p$. $4s$ and $4p$ are actually replaced by equivalent screenings by Fermi electrons (cf. § 9).

The author is greatly indebted to Professor N. F. Mott, who suggested the present problem, for his constant help and encouragement during the investigation. He wishes also to thank Dr. D. Polder and Mr. J. S. Plaskett for stimulating discussions.

APPENDIX

(1) DISPLACEMENT OF CHARGE IN A FERMI GAS

Let $V(r)$ be a spherically symmetrical potential such that $r^2V(r)$ tends to zero at the origin and at infinity. We denote by ψ_l and ψ'_l two solutions of the corresponding Schrödinger equation for two different values E and E' of the energy

$$\begin{aligned}\frac{\partial^2 \psi_l}{\partial r^2} + \left[2(E - V) - \frac{l(l+1)}{r^2} \right] \psi_l &= 0, \\ \frac{\partial^2 \psi'_l}{\partial r^2} + \left[2(E' - V) - \frac{l(l+1)}{r^2} \right] \psi'_l &= 0.\end{aligned}$$

We multiply the first equation by ψ'_l , the second by ψ_l and subtract; then

$$2(E' - E)\psi_l\psi'_l = \psi'_l \frac{\partial^2 \psi_l}{\partial r^2} - \psi_l \frac{\partial^2 \psi'_l}{\partial r^2}.$$

An integration gives

$$2(E' - E) \int_0^R \psi_l \psi'_l dr = \left[\psi'_l \frac{\partial \psi_l}{\partial r} - \psi_l \frac{\partial \psi'_l}{\partial r} \right]_0^R.$$

Let now the energies tend towards the same value

$$E' \rightarrow E = \frac{1}{2}k^2.$$

We may replace ψ'_l by $\psi_l + (\partial \psi_l / \partial k) dk$, and $(E' - E)$ by $k dk$. This substitution gives, after dividing both sides by dk

$$2k \int_0^R \psi_l^2 dr = \left[\frac{\partial \psi_l}{\partial k} \frac{\partial \psi_l}{\partial r} - \psi_l \frac{\partial^2 \psi_l}{\partial k \partial r} \right]_0^R.$$

We replace now ψ_l by its asymptotic values for 0 and R . When $r \rightarrow 0$, we have $\psi_l \sim A r^{l+1}$ (A a constant). When $r \rightarrow \infty$, we have $\psi_l \sim \sin[kr + \eta_l(k) - \frac{1}{2}l\pi]$ (η_l the phase shift). Hence, when R tends to infinity

$$2k \int_0^R \psi_l^2 dr \simeq k \left(R + \frac{d\eta_l}{dk} \right) - \frac{1}{2} \sin 2(kR + \eta_l - \frac{1}{2}l\pi).$$

When potential V vanishes, we obtain a Bessel function

$$2k \int_0^R \frac{1}{2} \pi J_{l+\frac{1}{2}}^2(kr) dr \simeq kR - \frac{1}{2} \sin 2(kR - \frac{1}{2}l\pi).$$

A subtraction leads therefore to the equality

$$\frac{2}{\pi} \int_0^R [\psi_l^2 - \frac{1}{2} \pi k r J_{l+\frac{1}{2}}^2(kr)] dr \simeq \frac{1}{\pi} \frac{d\eta_l}{dk} + \frac{1}{k\pi} \cos \eta_l \sin (2kR + \eta_l - l\pi).$$

The left member of this equation gives the amount of charge attracted by potential V in the l th spherical component of an electronic state with momentum k in a Fermi gas. The second term on the right is a sinusoidal function of zero average value. This gives no contribution if R tends to infinity; and we have

$$\frac{2}{\pi} \int_0^\infty [\psi_l^2 - \frac{1}{2} \pi k r J_{l+\frac{1}{2}}^2(kr)] dr = \frac{1}{\pi} \frac{d\eta_l}{dk}.$$

This reasoning uses an approximate sinusoidal form for ψ_l at infinity. Using the WKB approximation, one can show that the terms neglected do not alter the final result.

(2) WANNIER-SLATER'S THEOREM

We make the following definitions: V_L is the potential of a perfect lattice and V_p a perturbing potential, equal to zero at infinity. $\mathbf{k}$ being an arbitrary constant vector, $\psi_1 = \phi_1 \exp(i\mathbf{k} \cdot \mathbf{r})$ is a wave function solution of the perfect lattice for the energy E and $\phi_2 \exp(i\mathbf{k} \cdot \mathbf{r})$ is the wave function of an electron with momentum $\mathbf{k}$ in potential V_p . A simple mathematical derivation gives

$$\nabla^2[\phi_1 \phi_2 \exp(i\mathbf{k} \cdot \mathbf{r})] + 2[E - V_L - V_p] \phi_1 \phi_2 \exp(i\mathbf{k} \cdot \mathbf{r}) = 2(\nabla \phi_1 \cdot \nabla \phi_2) \exp(i\mathbf{k} \cdot \mathbf{r}).$$

$\phi_1 \phi_2 \exp(i\mathbf{k} \cdot \mathbf{r})$ is therefore an approximate solution in the perturbed lattice if the potential $(\nabla \phi_1 \cdot \nabla \phi_2)(\phi_1 \phi_2)^{-1}$ given by the corrective term is small. When nothing is known about V_p the average value of the corrective energy is given by $\int_V \phi_1^* \nabla \phi_1 d\tau$ which is easily proved to be equal to

$$\int_S \left[\psi_1^* \frac{\partial}{\partial n} (\mathbf{r} \psi_1) - \psi_1 \mathbf{r} \frac{\partial}{\partial n} \psi_1^* \right] d\sigma - i\mathbf{k} \int_V \phi_1^* \phi_1 \exp(-i\mathbf{k} \cdot \mathbf{r}) d\tau.$$

Both terms disappear in this expression if ϕ_1 is periodical in $\mathbf{r}$. The best choice for $\mathbf{k}$ is therefore such that ϕ_1 is a Floquet function.

We may therefore state the following: for an electron of given energy in a perturbed lattice, we denote by ϕ_1 the Floquet function in the perfect lattice corresponding to the same energy and by $\mathbf{k}$ the corresponding momentum: the wave function of the electron is approximately the product of ϕ_1 and the wave function $\psi_2 = \phi_2(i\mathbf{k} \cdot \mathbf{r})$ of the electron of momentum $\mathbf{k}$ travelling in the perturbing potential.

The corrective potential $(\nabla\phi_1 \cdot \nabla\phi_2)(\phi_1\phi_2)^{-1}$ is small in an s band and for perturbing potentials V_p possibly Coulombian at the origin but varying slowly at large distances. Near the edge of a band, ϕ_1 may be replaced by the wave function at the bottom of the band, with the use of an effective mass, as in the Wigner-Seitz method.

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XV. *Some Features of Nuclear Disintegrations Caused by Cosmic Rays*

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SUMMARY

Cosmic ray stars in light and heavy elements have been investigated by the photographic plate method. The angular distribution of the particles from stars in heavy elements is shown to be isotropic, while that of those from light elements is not. Possible explanations of this are discussed. The numbers of stars with various numbers of alpha particles and protons are shown to be in accordance with a random distribution. This indicates that the emission of one charged particle has no influence on the nature of a subsequent charged particle. The emission of excited beryllium eight nuclei is also investigated and their frequency found to agree with the evaporation theory.

INTRODUCTION

It has been shown by a number of investigators (Harding, Lattimore and Perkins 1948, Le Couteur 1950, Page 1950, Hodgson 1950 a, b) that the main features of the disintegration of nuclei after they have been highly excited by collisions with energetic particles of the cosmic radiation may be adequately accounted for by the evaporation theory.

Phenomena have, however, been recorded (Harding, Lattimore and Perkins 1948, § 5; Perkins 1950 a) which do not agree with the evaporation theory. In this paper an account is given of a search for such anomalies in the angular distribution of the particles (Part I) and in the number of particles of various types (Part II). Part III is devoted to an investigation of the possibility of emission of unstable beryllium eight excited to various energy states.

EXPERIMENTAL PROCEDURE

The observations were made on disintegrations in Ilford C2 emulsion exposed horizontally to the cosmic radiation on the Jungfraujoch and processed in the usual manner. C2 emulsion was chosen rather than the more sensitive G5 so that the tracks of alpha particles could be readily distinguished from those of protons. The energetic particles emitted in the initial impact of the cosmic ray particle with the target nucleus are not therefore considered here, but only the lower energy particles emitted

* Communicated by Sir George Thomson, F.R.S.

during the subsequent de-excitation. Stars so near to the surface of the emulsion that one or more tracks could not be identified were omitted from the analysis.

Throughout this work, all singly charged particles were included in the class of protons and all particles of charge greater than two in the class of alpha particles.

Particles crossing the emulsion sometimes collide with hydrogen nuclei, producing a forked track which simulates a three track star. The relation between the spatial configuration of the tracks and the masses of the particles enabled them to be recognized and excluded from the analysis.

The stars were classified according to their numbers of tracks, and, for those of less than seven tracks, according as they had a recoil or not. A recoil is a short, thick track due to the residual nucleus. Tracks less than four microns in length were counted as recoils.

Stars of more than six tracks are nearly all due to the disintegration of the heavy elements silver and bromine. Of the stars with six tracks or less, Harding (1949) has shown that those with recoil fragments are mainly due to the heavy elements also. The majority of the small stars without recoils are formed in the light elements carbon, nitrogen and oxygen. The remainder are stars in the heavy elements for which the recoil is invisible owing to its short range or unfavourable orientation. These considerations enable a partial separation of the disintegrations in light and heavy elements to be made.

PART I—THE ANGULAR DISTRIBUTION

§1. INTRODUCTION

The evaporation theory predicts that particles will be emitted at random in the centre-of-mass system of the excited nucleus. It is of interest to examine the extent to which this is fulfilled for nuclear disintegrations produced by cosmic rays.

In an investigation of the angular distribution of emitted particles, it is important to distinguish between the departures from isotropy caused by a movement of the emitting nucleus with respect to the apparatus, and those caused by an anisotropic mode of disintegration in the centre of mass system. These two sources of anisotropy will be referred to as extrinsic and intrinsic respectively.

The extrinsic angular distribution of the particles emitted from nuclear disintegrations caused by cosmic rays has been investigated using photographic emulsions by Brown, Camerini, Fowler, Heitler, King and Powell (1949), by Bernardini, Cortini, and Manfredini (1950) and by Manfredini (1951). They measured the angular distribution of the star particles projected on a vertical plane and found that the fast particles emitted in the initial impact were strongly collimated downwards. Those lower energy particles emitted during the subsequent de-excitation process

were slightly collimated downwards in the stars of more than six tracks, and more strongly collimated downwards for the smaller stars.

This downward collimation of the evaporated particles can be attributed to the motion of the excited nucleus. The increase in collimation for the small stars is in accordance with expectation since, for the same momentum transfer, a light nucleus will be given a greater downward velocity than a heavy one.

Perkins (1950 a) has used the numbers of alpha particles and protons emitted upwards and downwards from large stars to calculate the downward velocity of the excited nucleus. For stars of up to eleven tracks he found that the downward velocities calculated separately from the alpha particles and the protons are the same. This is what would be expected if all the particles were emitted isotropically in the centre of mass system. For stars of more than eleven tracks, however, there was a marked discrepancy, which he attributed to an anomaly in the angular distribution of the alpha particles due to an abnormal mode of disintegration which has not yet been explained.

In order to examine the mechanism of disintegration, the intrinsic angular distribution of the star particles has been investigated by several authors using different methods.

Perkins (1948) and Harding, Lattimore and Perkins (1949) have associated a unit vector with each track and evaluated the resultant for each star. The observed distribution of resultant vectors was then compared with the distribution expected on Chandrasekhar's random walk theory (1943). A significant difference between the two distributions indicates anisotropy. This was found only for alpha particles from very high energy disintegrations in which fifteen or more charged particles were emitted. However, this method is laborious since it involves measuring the orientation of every track in the star and calculating the resultant of unit vectors laid along them.

Another method, developed by Lovera (1949), consists in measuring the distribution of all the angles, projected on the plane of the emulsion, between adjacent pairs of star particles. If the particles are randomly emitted from a massive nucleus, the theoretical distribution of these angles, for an n -track star, is given by

$$P(\phi)d\phi = \frac{n-1}{2\pi} \left(1 - \frac{\phi}{2\pi}\right)^{n-2} d\phi, \quad 0 \leq \phi \leq 2\pi.$$

A variant of this method, developed independently at Imperial College, consists in measuring the least projected angle ζ , on the plane of the emulsion, between any two tracks of the star. The corresponding theoretical distribution for random emission is given by

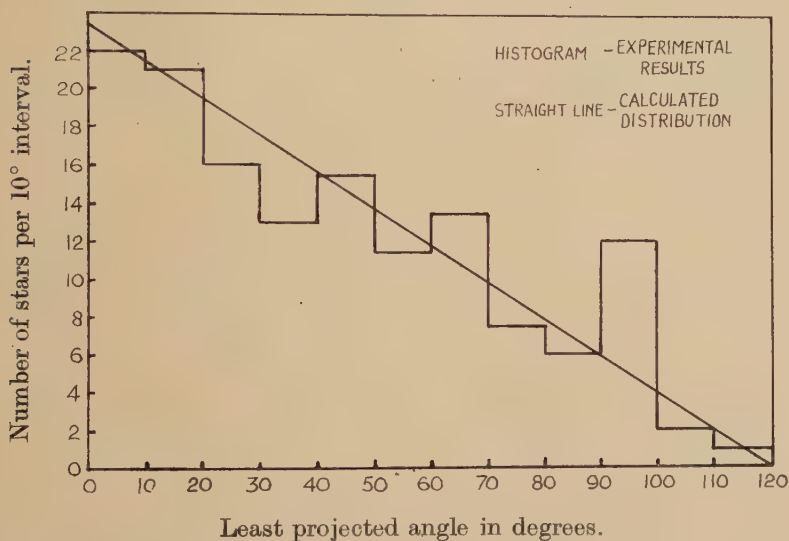
$$P(\zeta)d\zeta = \frac{n(n-1)}{2\pi} \left(1 - \frac{n\zeta}{2\pi}\right)^{n-2} d\zeta \quad \zeta < \frac{2\pi}{n}$$

$$= 0, \quad \zeta > \frac{2\pi}{n}.$$

In the following sections the results of measurements of this quantity are described and discussed.

This method, like the others described here, does not provide any *a priori* way of distinguishing between intrinsic and extrinsic anisotropy. This must be done from a consideration of the conditions of exposure or of the type of disintegration studied. (See §3.) Thus, for example, extrinsic anisotropy would have a much greater effect on the distribution of ζ for stars on plates exposed vertically than for those on plates exposed horizontally, whereas little difference between the two distributions would be expected from intrinsic anisotropy. Comparison of groups of stars on plates exposed in both ways can therefore be used to distinguish between the two kinds of anisotropy.

Fig. 1



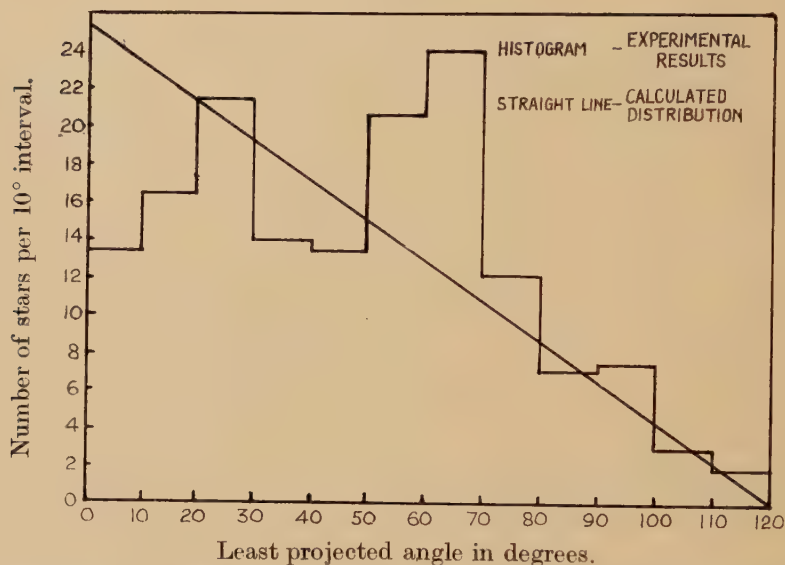
Distribution of least projected angle for three track stars with recoil fragments.

§ 2. EXPERIMENTAL RESULTS

The least projected angle was first measured, considering all the tracks, for each star in a randomly selected group of a thousand stars. The stars were then grouped according to their numbers of protons, and the least projected angle measured for each, only the proton tracks being considered. This was repeated considering only the alpha particle tracks. Thus a particular star can be included in up to three groups. The stars were classified according to the number of tracks N considered in the measurement. The distributions of least projected angles for the various groups of stars were compared with those expected on random emission. Typical results, for three track stars (with and without recoils), are shown in figs. 1 and 2.

The agreement between the two distributions was examined in each case by the χ^2 test, and the probabilities that the observed distributions are chance fluctuations on the theoretical are given in table 1. The results in the columns headed ' N proton tracks' refer to measurements in which only the proton tracks were considered, and similarly for the columns headed ' N alpha tracks'. Thus the three measurements on a five-track star of three protons and two alpha particles appear in three columns of the table. The results for two alpha tracks only refer to measurements on stars with just two alpha particles, irrespective of their numbers of protons, and similarly for the other groups. Results for the large stars are omitted because of the small number available.

Fig. 2



Distribution of least projected angle for three track stars without recoil fragments.

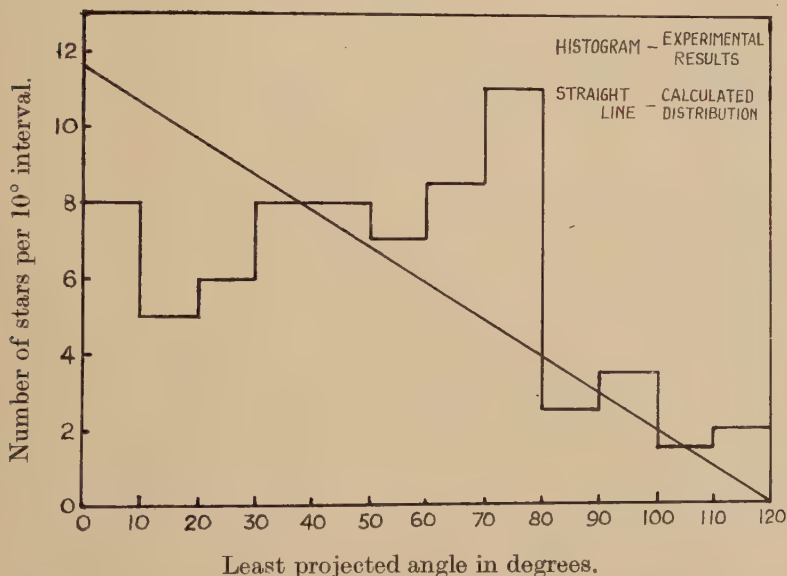
Table 1. Probabilities, according to the χ^2 Test, that the Observed Distributions of ζ are Random Fluctuations from the Theoretical Distributions

Number of tracks N	Stars without recoils			Stars with recoils			π^- -meson stars (all tracks)
	All N tracks	N proton tracks	N alpha tracks	All N tracks	N proton tracks	N alpha tracks	
2	—	0.06	0.02	—	0.3	0.8	0.1
3	0.003	0.2	0.01	0.6	0.7	0.9	0.008
4	0.2	0.2	—	0.9	0.5	0.2	0.07
5	0.1	0.2	—	0.7	0.4	—	—
6	0.2	—	—	0.5	—	—	—

These results show that the agreement between theory and experiment is satisfactory for those stars with recoils. Therefore the particles from the disintegrations of heavy nuclei are randomly emitted, in agreement with the evaporation theory.

The probabilities for the small stars without recoil fragments, which are mainly the disintegrations of light elements, are in many cases less than 0.05. This is commonly considered to indicate a significant difference between the two distributions. Since emission from stars in heavy elements has been shown to be random, this result shows that emission from the stars in light elements alone is even more anisotropic than from the recoil-less stars considered as a whole.

Fig. 3



Distribution of least projected angle for three track stars initiated by π^- -mesons.

This conclusion is supported by the results of similar measurements on two-, three-, and four-track stars initiated by π^- -mesons, which are nearly all formed in light elements. (Perkins 1949, Menon, Muirhead and Rochat 1950.) These results are included in table 1, and the distribution of least projected angle for three-track π^- -meson stars given in fig. 3. These were shown by the χ^2 test to be consistent with the corresponding distributions for small stars without recoils.

The results show no significant difference in angular distribution between alpha particles and the protons.

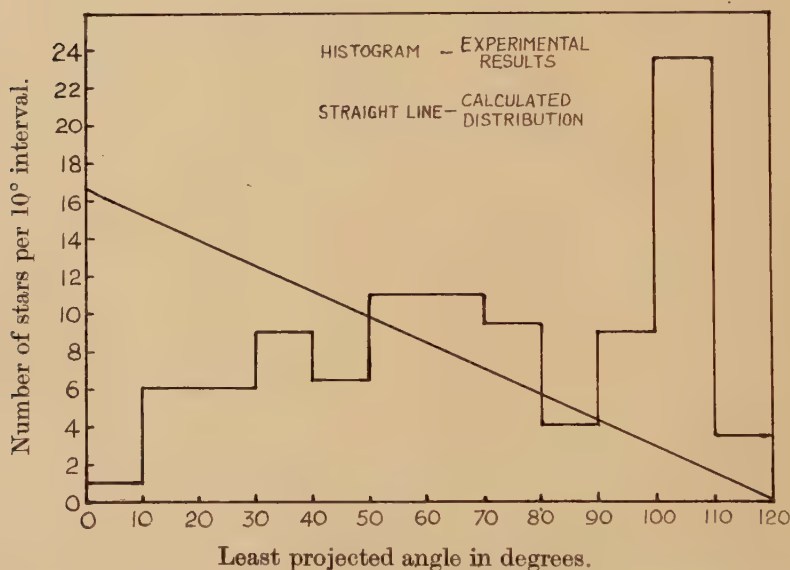
Possible explanations of this anisotropy of particles emitted from stars formed in light elements are discussed below.

§ 3. DISCUSSION

The anisotropy cannot be ascribed to motion of the excited nucleus, since this would increase the number of stars with small least projected angles, contrary to what is observed. Also, since the plates used in the present experiment were exposed horizontally, motion of the excited nucleus could only have a small effect on the least projected angle distribution. The effect would be zero if all the star producing radiation was incident vertically. Therefore the observed anisotropy must be intrinsic, and due to the mode of disintegration of the light elements.

In order to see whether stars produced in one specific nuclear reaction would show an anomalous distribution the method was applied to the ($^{12}\text{C}(\gamma, 3\alpha)$) stars observed by Goward and Wilkins (1951), who kindly sent their measurements. The results are plotted in fig. 4, and indicate

Fig. 4



Distribution of least projected angle for three track $^{12}\text{C}(\gamma, 3\alpha)$ stars observed by F. K. Goward and J. J. Wilkins.

a marked anisotropy. This supports the view that the emission of particles from a light nucleus is anisotropic. The results for small cosmic ray stars of course represent the average over a large number of different reactions. It is likely that many of the anisotropic features cancel out when such a group of stars is considered, but, nevertheless, some evidence of anisotropy remains.

A possible explanation of this is that a particle is often emitted when the nucleus is recoiling from the emission of a previous one. This causes the angle between the directions of motion of the two particles to be larger in the laboratory system than in the centre-of-mass system, and this

increases the least projected angle. The effect is quite small owing to the disparity between the velocities of recoil and emission, and also because a neutron must often intervene between successive charged particles, destroying any angular correlation between them.

The observations may also be accounted for by assuming that a light nucleus, when struck by an energetic particle, is shattered in one act, instead of evaporating via a number of intermediate nuclei as it does when struck by a low energy particle. The mutual obstruction of the particles would cause them to be more symmetrically emitted than in the case of random evaporation. This also would result in an excess of stars with large least projected angles.

It is not possible to choose between these alternatives without a detailed calculation of the effect of the nuclear recoil on the distribution of least projected angles. This calculation is now in progress and will be reported on completion.

PART II—THE FREQUENCIES OF STARS WITH VARIOUS NUMBERS OF ALPHA PARTICLES AND PROTONS

§ 1. INTRODUCTION

Several authors (Perkins 1950 b, Crussard 1950 a, b, Hodgson 1950) have shown that pairs or triplets of particles due to the emission and subsequent disintegration of unstable fragments are sometimes observed in nuclear disintegrations caused by cosmic rays. In general these fragments are only seen if their energy of break-up is small, since then the disintegration products continue nearly in the direction of the original fragment, giving a narrow cone of particles which is readily recognizable. If, on the other hand, the energy of break-up is large, the disintegration products are widely separated and are lost in the background of evaporated particles. It is therefore of interest to examine the distribution of alpha particles and protons among the stars for any anomalies which might be ascribed to unstable fragments.

If the emission of one particle does not affect the probability of emission of another similar particle, then the frequencies of emission will follow Bernoulli's Law. Thus, if p_α is the probability of emission of an alpha particle from a disintegration in which n charged particles are emitted, then the proportion of a large group of such stars with just m alpha particles is

$$\frac{n!}{m!(n-m)!} p_\alpha^m (1-p_\alpha)^{n-m} \quad p_\alpha = [1 - (\alpha/p)^{-1}]^{-1},$$

where (α/p) is the ratio between the numbers of doubly and singly charged particles emitted from the disintegrations considered.

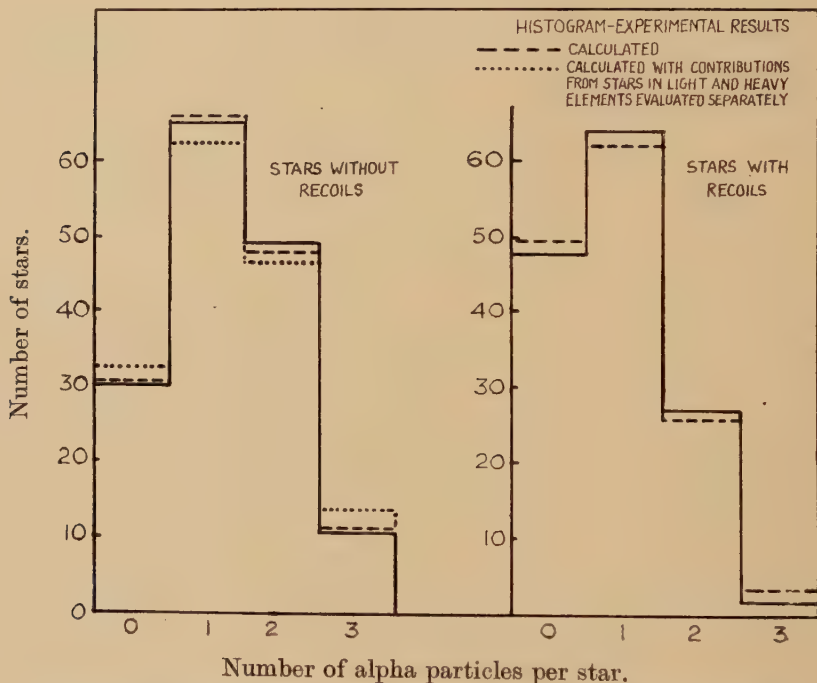
§ 2. COMPARISON OF RESULTS WITH THEORY

One thousand stars were examined and the numbers of alpha particles and protons in each recorded.

The distribution given by the above formula were calculated for each group of stars and compared with the experimental by the χ^2 test. Typical results, for three-track stars with and without recoils, are shown in fig. 5. The probability in each case that the observed distribution is a random fluctuation on the theoretical is given in table 2.

Since stars without recoils are a mixture of stars in light and stars in heavy elements, the relevant calculations should take this into account, and the sum of two terms of the above form with the α/p ratios of stars in the light and heavy elements evaluated. Using approximate values

Fig. 5



Distribution of alpha particles and protons among three track stars..

Table 2. Probabilities, According to the χ^2 Test, that the Observed Distributions of Particles among the Stars are Random Fluctuations from the Theoretical Distributions

Star size	Stars without recoils	Stars with recoils
3	0.9	0.7
4	0.2	0.1
5	0.1	0.8

of the α/p ratios and of the relative proportion of the stars in light and heavy elements, this calculation was made, and the results are shown in fig. 5 by the dotted lines. It can be seen that this refinement makes little difference to the final result.

Table 2 shows that in no case is there a significant difference between the observed and calculated distributions. These results therefore give no support to the theory that some alpha particles come from the disintegration of excited fragments and consequently tend to come out in pairs or triplets, but, of course they do not exclude the possibility of the occurrence of such fragments in a small number of cases.

PART III—THE EMISSION OF BERYLLIUM EIGHT FROM NUCLEAR DISINTEGRATIONS

Perkins (1950 b) and Crussard (1950 a, b) have observed close pairs of alpha particles emitted from nuclear disintegrations caused by cosmic rays. Their frequency of occurrence is greater than would be expected from random emission, and they are ascribed to the emission of ${}^8_4\text{Be}$ nuclei which almost immediately decay into two alpha particles.

The energy of disintegration of the beryllium nucleus can be calculated from the energies of the two alpha particles and the angle between their directions of emission. The values obtained are about 90 kev in agreement with those obtained by other investigators (Hemmedinger 1948, 1949; Tollestrup, Fowler and Lauritsen 1949) using artificially produced beryllium nuclei.

In order to extend this work to other possible excited states of beryllium eight, measurements were made on all the pairs of alpha particles, both of which ended in the emulsion, from five hundred disintegrations. The energy of excitation, assuming the alpha particles to be from the disintegration of beryllium eight, was calculated for each pair. The distribution of energies of excitation was compared with that expected if all the particles were randomly evaporated. If any of the alpha-particle pairs are due to the emission and subsequent disintegration of beryllium eight, then the corresponding energy of excitation should appear as a peak on this distribution. It was found that the two distributions were consistent, except for an excess of eight pairs with small excitation energies due to the ground state of beryllium eight. Correcting for those pairs passing out of the emulsion, this gives a mean frequency of emission of 0.024 ± 0.008 , in good agreement with the approximate value 0.02 calculated by Le Couteur (1950 b) on the evaporation theory. The mean value of their disintegration energies is 82 kev, in good agreement with the results of other authors. The results give no evidence for the emission of beryllium nuclei excited to higher states, but they do not exclude their occurrence in a small number of cases.

Recently, Millar and Cameron (1951) have reported similar events in which the track of the beryllium nucleus is visible. They find a value of $(5.3 \pm 1.1) \times 10^{-14}$ sec for its lifetime. Wilkins and Goward (1951),

however, have examined their stars and find that it is only possible to put an *upper* limit to the lifetime of about 5×10^{-14} sec. The more energetic particles emitted from the disintegrations considered in this paper were therefore examined, and the distance from the star centre to where the two alpha particle tracks were resolved was measured and divided by the velocity of the particles as determined from their ranges. These results, in agreement with those of Wilkins and Goward, showed that it is only possible to put an upper limit of about 2×10^{-14} sec on the lifetime of beryllium eight.

ACKNOWLEDGMENTS

I should like to express my thanks to Professor Sir George Thomson, Mr. S. Lattimore and Mr. R. Elston for many valuable discussions, to Mr. F. K. Goward and Mr. J. J. Wilkins for communicating the results of their measurements, and to Dr. K. J. Le Couteur for communicating the result of his calculation on the emission frequency of beryllium eight.

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XVI. *The Non-Hookean Interaction of a Dislocation with a Lattice Inhomogeneity*

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ABSTRACT

The equilibrium position of an edge dislocation near certain types of lattice irregularities is investigated quantitatively, using the Peierls model of a dislocation. The external shear stress required to maintain equilibrium is calculated and the result extended to an array of like dislocations piled up against a barrier. The integral equation for the atomic displacements is linearized by a perturbation technique, the resulting equation being solved exactly.

§1. INTRODUCTION

It is well known that dislocations in a crystal lattice interact with each other, and with other structural defects and discontinuities. Of these, commonly occurring examples are solute atoms, vacant lattice sites, grain and crystal boundaries, micro-cracks and changes in elastic constants or lattice parameter. Interaction effects can be of great importance for the understanding of the mechanical properties of crystals, as is shown, for instance, by the work of Taylor (1934), Cottrell (1949), Mott and Nabarro (1947) and Seitz (1950). The main purpose of the present paper is to attempt to give a quantitative analysis of the effect on a dislocation of certain types of barriers and discontinuities which are thought to exist in actual crystals. More precisely, we set up a general method for calculating the external shear stress τ required to hold a dislocation in equilibrium at a given distance s from a specified lattice inhomogeneity.

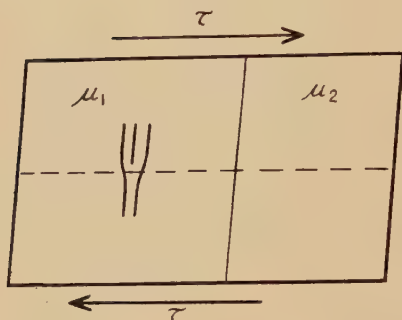
At the very outset of the work it is necessary to exclude from consideration all barriers which involve a discontinuity in elastic constants, for these have so far proved to be mathematically intractable.† Thus, for instance, it appears to be quite impossible to calculate $\tau(s)$, even approximately, for the approach of a dislocation towards an ideal surface of separation in a bicrystal as illustrated in fig. 1. Fortunately, this type of barrier is not of very great interest from the point of view of mechanical properties. Most of the inhomogeneities of

* Communicated by Professor H. Jones.

† Koehler (1941) has analysed the equilibrium of a dislocation in a finite single crystal, and since this is an extreme case of a bicrystal ($\mu_1 > 0$, $\mu_2 = 0$) his method may possibly be generalized.

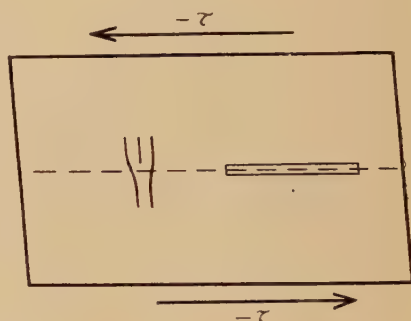
physical interest can be effectively represented by a model in which there is a change in the interatomic law of force across the slip plane, but no change of elastic constants elsewhere in the material (which is assumed to be elastically isotropic). This means that whilst there occurs an alteration in the nature of the 'glue' which binds together the two dislocated half-crystals, there is no *effective* alteration elsewhere. A direct example

Fig. 1



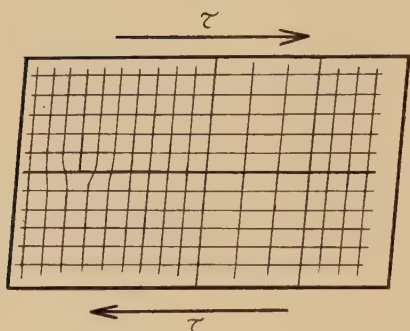
The rigidity modulus jumps from μ_1 to μ_2 on crossing the barrier.

Fig. 2



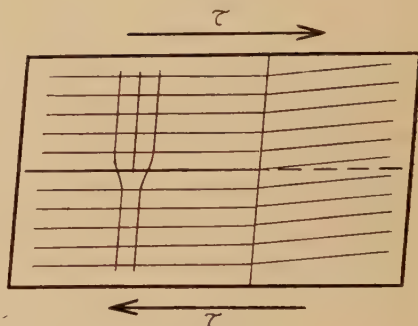
A micro-crack in the slip plane; the external shear stress τ is negative in this case, for the crack attracts the dislocation.

Fig. 3



A change in lattice parameter blocking the path of a dislocation.

Fig. 4



A dislocation near a grain boundary. The broken line represents the hypothetical extension of the slip plane.

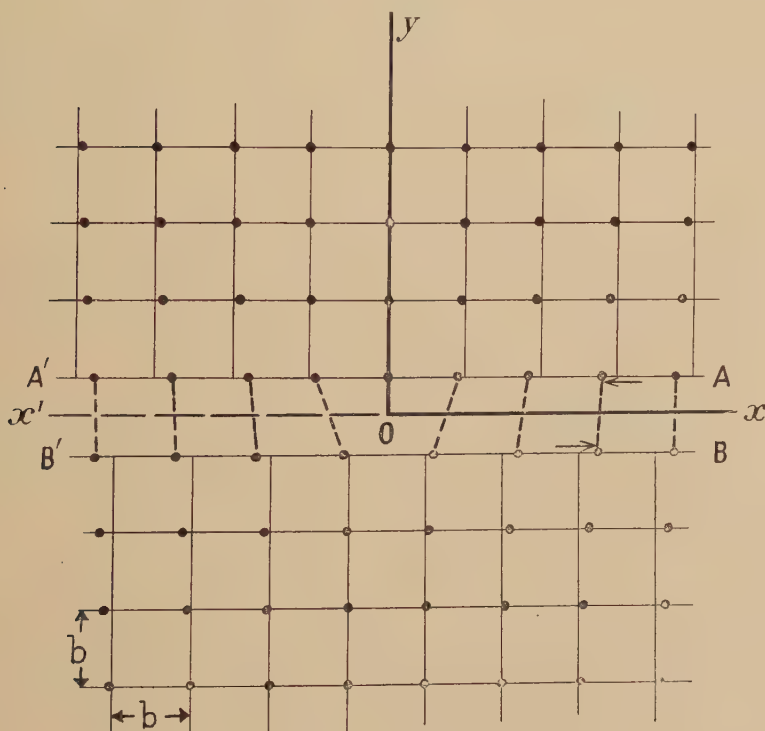
of such a situation is provided by a Zener micro-crack (1946), this being defined as a region of the slip plane across which there exists no law of force between the atoms (fig. 2). Another example is provided by a material in which there is a change of lattice parameter over a certain region, as illustrated in fig. 3. A third and important type of inhomogeneity is provided by a grain or twin boundary. A dislocation

is obviously repelled from such a barrier, and this can be regarded as being effectively due to a 'hardening of the 'glue' across a hypothetical extension of the slip plane (fig. 4). The barrier is defined mathematically as the point on the slip direction at which the law of force across the slip plane alters.

§2. THE PEIERLS MODEL OF A DISLOCATION

In order to calculate the interaction between dislocations and barriers, we adopt the Peierls model of a dislocation (Peierls 1940, Nabarro 1947), in which the atomic structure of the lattice is taken into account along the

Fig. 5



Cross-section of a positive edge dislocation.

slip plane. Applied to a dislocation in a homogeneous single crystal, this is successful in leading to stresses and strains which do not diverge in the neighbourhood of the origin, and which at the same time approximate to those of the Burgers classical dislocation elsewhere. The extension of this analysis to the case where the atomic structure along the slip plane is no longer constant but varies in a specified manner, constitutes the essential mathematical problem of the present paper.

A diagram of the cross-section of an edge dislocation is shown in fig. 5, the elastic problem being a two dimensional one of plane strain.

The dislocation is constructed by cutting the material along $x'Ox$, inserting an extra row of atoms along Oy and applying a system of equal and opposite shear stresses to the surfaces $A'A$ and $B'B$ until the corresponding atoms in each move as far as possible into alignment. These surface stresses are the horizontal component of the interaction forces between corresponding atoms in the two surfaces, the normal component being neglected.

The displacement $u(x)$ of an atom in $A'A$ will be approximately equal and opposite to that of the corresponding atom in $B'B$, so that their relative displacement r is given by

$$\left. \begin{aligned} r(x) &= 2u(x) + b/2, & x > 0, \\ r(x) &= 2u(x) - b/2, & x < 0, \end{aligned} \right\} \quad \dots \dots \dots (1)$$

the x -axis being the direction of positive $u(x)$. If for convenience we adopt the non-dimensional measure of displacement ψ , where

$$\psi = 2u/b, \quad \dots \dots \dots (2)$$

then (1) becomes

$$r(x)/b = \psi(x) \pm \frac{1}{2}. \quad \dots \dots \dots (3)$$

For a dislocation in equilibrium in a homogeneous single crystal there must be perfect alignment of corresponding atoms at infinity, so that $r(\pm\infty) = 0$, leading to

$$\psi(\infty) = -\psi(-\infty) = -\frac{1}{2}. \quad \dots \dots \dots (4)$$

If the crystal is not homogeneous then an external shear stress τ is required to maintain equilibrium. The shear stress in the slip plane accordingly reduces to τ at $\pm\infty$, so that if this is small, we have by Hooke's law $r(\pm\infty)/b = \tau/\mu$ leading from (3) to

$$\left. \begin{aligned} \psi(\infty) &= -\frac{1}{2} + (\tau/\mu), \\ \psi(-\infty) &= \frac{1}{2} + (\tau/\mu), \end{aligned} \right\} \quad \dots \dots \dots (5)$$

where μ is the shear modulus.

Assuming the two half-planes are isotropic elastic continua,* the relation between the shear stress $p(x)$ and du/dx along $A'A$ is given by

$$p(x) = \frac{\mu}{\pi(1-\nu)} \int_{-\infty}^{\infty} \frac{du(t)}{dt} \frac{dt}{t-x} + \tau, \quad \dots \dots \dots (6)$$

where ν is Poisson's ratio (Nabarro 1947); the Cauchy principal value is taken at $t=x$. If distances x, y, s are measured in units of β , where $\beta = b/2(1-\nu)$ and the notation (2) is adopted, then (6) becomes

$$p(x) = \frac{\mu}{\pi} \int_{-\infty}^{\infty} \frac{d\psi(t)}{dt} \frac{dt}{t-x} + \tau, \quad \dots \dots \dots (7)$$

* The present theory may be easily modified for an anisotropic crystal, following the method of Eshelby (1949).

it being noted that $p(\pm\infty)=\tau$. An alternative integral relation, which is quite equivalent to (7) is

$$\frac{d\psi(x)}{dx} = -\frac{1}{\mu\pi} \int_{-\infty}^{\infty} \{p(t)-\tau\} \frac{dt}{t-x}, \quad \dots \dots (8)$$

this being determined either as the Hilbert transform (Titchmarsh 1948) of (7), or again directly from elasticity theory (Leibfried and Lucke 1949). Since the shear stress distribution is provided by the interatomic forces across the slip plane, it follows that p is a function of r and therefore of ψ , i.e. $p(x)=f[\psi(x)]$, where f is a function either known or assumed. On now substituting for $p(x)$ in either (7) or (8), we obtain the integral equation required.

The function $p(r)$ has not yet been calculated, but over any homogeneous region of the crystal it must be periodic with period b and alternately positive and negative in each half-period. Furthermore, it must everywhere have the limiting form $p=\mu r/b$ for small relative displacements, in accordance with Hooke's law. A simple function of this type, viz.,

$$p=(\mu/2\pi) \sin 2\pi(r/b) = -(\mu/2\pi) \sin 2\pi\psi \quad \dots \dots (9)$$

was put forward by Peierls as the most convenient assumption as to the law of force in a homogeneous single crystal. On setting this into (7), we obtain the integral equation

$$-\frac{\mu}{2\pi} \sin 2\pi\psi(x) - \tau = \frac{\mu}{\pi} \int_{-\infty}^{\infty} \frac{d\psi(t)}{dt} \frac{dt}{t-x}, \quad \dots \dots (10)$$

which, although non-linear and falling into no standard form, has the exact solution

$$\psi(x) = -(1/\pi) \tan^{-1}x, \quad \dots \dots (11)$$

for $\tau=0$, as may be verified by substitution. Since this also satisfies the boundary condition (4), it represents a single dislocation in equilibrium in a crystal free from external stress. Another exact solution for non-zero τ is also known (Nabarro 1947), this representing a pair of dislocations of opposite sign held apart against their mutual attraction by the external shear stress.

If the crystal is not homogeneous, this means, as pointed out in the introduction, that the $p(r)$ function changes over the appropriate region of the slip plane. Thus, for instance, the presence of a Zener micro-crack of length l at the point s along the slip direction is represented by

$$\begin{aligned} p &= (\mu/2\pi) \sin 2\pi(r/b); & -\infty \leq x \leq s, \\ &= 0; & s < x < s+l, \\ &= (\mu/2\pi) \sin 2\pi(r/b); & s+l \leq x \leq \infty. \end{aligned}$$

As another example, a change in lattice parameter from b to b' at s means that

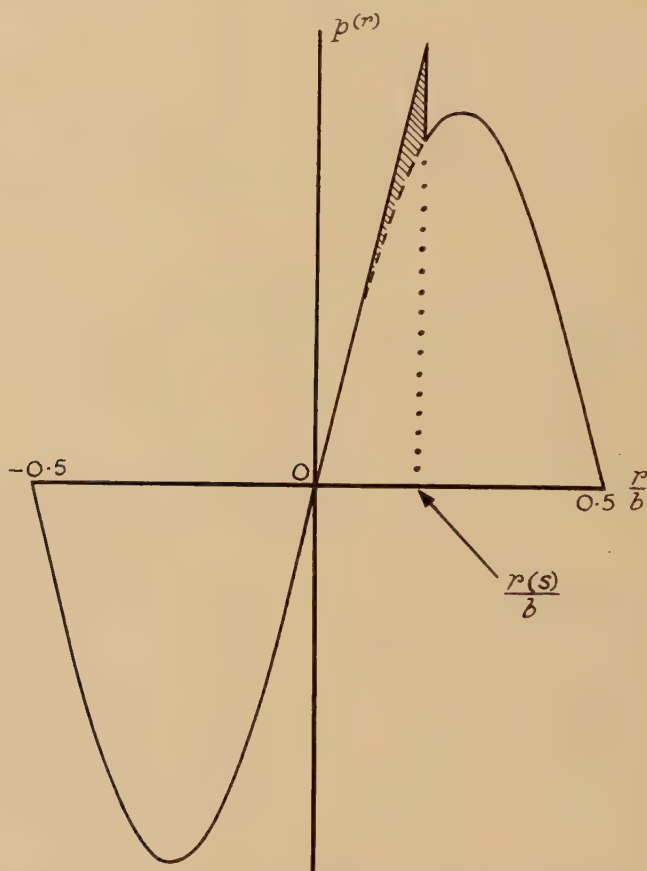
$$\begin{aligned} p &= (\mu/2\pi) \sin 2\pi(r/b); & -\infty \leq x \leq s, \\ &= (\mu/2\pi)(b'/b) \sin 2\pi(r/b'); & s < x \leq \infty. \end{aligned}$$

As a third example, the effect of a change in lattice orientation at s may be described by

$$p = (\mu/2\pi) \sin 2\pi(r/b); \quad -\infty \leq x < s, \\ = \mu r/b; \quad s < x \leq \infty.$$

This is plausible since the absence of periodicity on the extension of the glide plane is represented by a restoring force which increases steadily with relative displacement (Hooke's law instead of a periodic one).

Fig. 6



The general form of the function $p(r)$ when a dislocation lies near to a Hooke's law barrier.

The two laws of force in this case are illustrated by the discontinuous $p(r)$ function in fig. 6, $r = \pm(b/2)$ being the relative displacement at the origin (i.e. the centre of the dislocation), and $r = r(s)$ being the relative displacement at the barrier, where the law of force changes. A further example is provided by a solute atom lying in the slip plane. Owing to the difference in size and electronic structure between solute and solvent,

the solute will interact with the dislocation. The former effect has already been investigated (Cottrell 1950), whilst the latter can be dealt with to a rough approximation by the methods of this paper. Thus a solute atom* at $x=s$ can be regarded as a region along the slip plane such that

$$\begin{aligned} p(r) &= (\mu/2\pi) \sin 2\pi(r/b), & -\infty \leq x \leq s-(b/2), \\ &= (\mu'/2\pi) \sin 2\pi(r/b), & s-(b/2) < x < s+(b/2), \\ &= (\mu/2\pi) \sin 2\pi(r/b), & s+(b/2) \leq x \leq \infty, \end{aligned}$$

the parameter μ' being suitably chosen.

§3. THE EXTERNAL SHEAR STRESS

The integral equation for ψ may be written from (8) as

$$-\mu\pi \frac{d\psi}{dx} = \int_{-\infty}^{s_1} \frac{p[\psi(t)] - \tau}{t-x} dt + \int_{s_1}^{s_2} + \int_{s_2}^{\infty}, \quad . \quad . \quad . \quad (12)$$

where p is a given function of ψ for each separate region of the slip plane : more concisely,

$$\frac{d\psi}{dx} = -\frac{1}{\mu\pi} \int_{-\infty}^{\infty} \frac{p[\psi(t)] - \tau}{t-x} dt, \quad . \quad . \quad . \quad . \quad (13)$$

where the positions of the discontinuities are now parameters in the functional relation between p and ψ . It is convenient to take the Hilbert transform of (13), i.e.

$$p[\psi(x)] - \tau = \frac{\mu}{\pi} \int_{-\infty}^{\infty} \frac{d\psi(t)}{dt} \frac{dt}{t-x}, \quad . \quad . \quad . \quad . \quad (14)$$

which equation, together with the boundary conditions (5), will be adopted for the purposes of calculation.

A simple expression for the external shear stress τ may be derived as follows. Multiply through (14) by $d\psi/dx$ and integrate over all values of x to give

$$\int_{-\infty}^{\infty} (p - \tau) \frac{d\psi}{dx} dx = \frac{\mu}{\pi} I_1,$$

where

$$\begin{aligned} I_1 &= \int_{-\infty}^{\infty} \frac{d\psi}{dx} dx \int_{-\infty}^{\infty} \frac{d\psi}{dt} \frac{dt}{t-x} \\ &= - \int_{-\infty}^{\infty} \frac{d\psi}{dt} dt \int_{-\infty}^{\infty} \frac{d\psi}{dx} \frac{dx}{x-t} \\ &= -I_1, \end{aligned}$$

assuming that it is valid to interchange the order of integration. Since $I_1 = -I_1$, it follows that I_1 is zero, giving

$$\int_{-\infty}^{\infty} p \frac{d\psi}{dx} dx = \tau \left[\psi(x) \right]_{-\infty}^{\infty} . \quad . \quad . \quad . \quad . \quad (15)$$

* More correctly we should talk of a row of solute atoms in the slip plane and parallel to the dislocation line.

On now making use of the boundary conditions (5) on ψ , we have

$$\begin{aligned} -\tau &= \tau \left[\psi(x) \right]_{-\infty}^{\infty} = \int_{-\infty}^{\infty} p \frac{d\psi}{dx} dx \quad . \quad . \quad . \quad (16) \\ &= \int_{1/2+\tau/\mu}^{-1/2+\tau/\mu} p[\psi] d\psi \\ &= \int_{1/2+\tau/\mu}^0 p[\psi] d\psi + \int_0^{-1/2+\tau/\mu} p[\psi] d\psi, \\ &= \int_{\tau/\mu}^{-1/2} p d(r/b) + \int_{1/2}^{\tau/\mu} p d(r/b) \end{aligned}$$

from (3), i.e.

$$\tau = \int_{-1/2}^{1/2} p(r/b) d(r/b). \quad . \quad . \quad . \quad (17)$$

From the result* (17) it will be seen that the magnitude of the external shear stress is that of the shaded area shown in fig. 6. Although in this diagram the position $r(s)$ of the discontinuity is not known without calculation, it can usually be estimated in cases where the interaction is not too large. For example, if a Peierls dislocation lies outside a Hooke's law region, then it is a sufficiently good approximation to write

$$r(s) \simeq (b/\pi) \tan^{-1}(1/s), \quad . \quad . \quad . \quad (18)$$

this being the relative displacement at $x=s$ in the absence of the barrier. The resulting curve of τ against s , derived from the equation

$$\begin{aligned} \tau(s) &= \int_{-1/2}^{1/2} p\left(\frac{r}{b}\right) d\left(\frac{r}{b}\right) = \mu \int_0^{1/\pi \tan^{-1} 1/s} \left(\frac{r}{b} - \frac{1}{2\pi} \sin 2\pi \frac{r}{b}\right) d\left(\frac{r}{b}\right) \\ &= \frac{\mu}{2\pi^2} \left[\left(\tan^{-1} \frac{1}{s} \right)^2 - \frac{1}{1+s^2} \right], \quad (s \geq 0), \quad . \quad . \quad . \quad (19) \end{aligned}$$

is illustrated in fig. 7. For $s \gg 1$, $\tau(s) = \mu/6\pi^2 s^4$, i.e.

$$\tau(s) = \frac{\mu\beta^4}{6\pi^2 s^4} = \frac{\mu b^4}{96\pi^2 (1-\nu)^4 s^4}, \quad . \quad . \quad . \quad (20)$$

on reverting to ordinary units of length. This gives $s=5b$ for $\tau \sim 10^{-5}\mu$ ($\nu=1/3$), which means that a typical yield stress pushes the dislocation to within five interatomic spacings from the barrier. It is

* F. R. N. Nabarro has pointed out that this result may also be derived by the following virtual work argument. If the whole pattern of the displacement curve is moved bodily forward by $\delta\alpha$, i.e. the dislocation is moved forward, then the external shear stress does work $\tau b\delta\alpha$, which must be equal to the virtual change in the internal energy of the system. The strain energy of the two elastic continua (fig. 5) is unchanged, but the relative displacement of atoms in A'A and B'B changes by

$$\delta r = r(x - \delta\alpha) - r(x) = -\delta\alpha(d\psi/dx) = -b\delta\alpha(d\psi/dx),$$

so that the stresses $p(x)$ do work

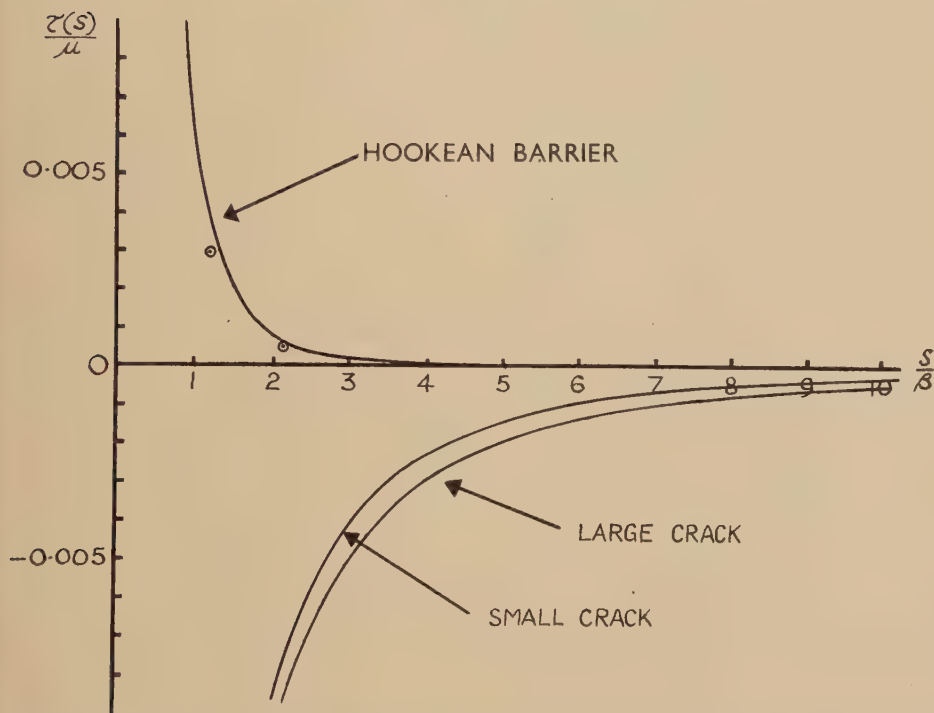
$$-b\delta\alpha \int_{-\infty}^{\infty} p(d\psi/dx) dx.$$

On equating this to $\tau b\delta\alpha$ we are led to (16), from which (17) is readily obtained.

interesting to note that when the centre of the dislocation coincides with the barrier, the relative displacement there is always $b/2$, in agreement with the result obtained by setting $s=0$ in (18), which estimate is in this instance therefore strictly correct. An immediate consequence is that $\tau(0)$ can always be calculated exactly, e.g. for a Hooke's law barrier $\tau(0)=(\mu/2\pi^2)[(\pi^2/4)-1]=7.4 \times 10^{-2}\mu$ on setting $s=0$ in (19).

It is obvious on physical grounds that, as illustrated in fig. 6, τ will increase* as s decreases, so that $\tau(s) \leq \tau(0)$. Whether the dislocation can proceed further and penetrate the barrier depends on the nature of

Fig. 7



Graph of the external shear stress τ required to hold a single dislocation at a distance s from a Hooke's law barrier, a large crack, and a small crack ($l=5\beta \simeq 4b$). The separate plots ○ are based on the more accurate values of $r(s)$, derived from the perturbation method.

the law of force in the region concerned. If it is not periodic with period b , then τ increases so rapidly that the dislocation is effectively blocked, as with the Hookean barrier above. If, on the other hand, the periodicity of the law of force is maintained, then τ attains its maximum

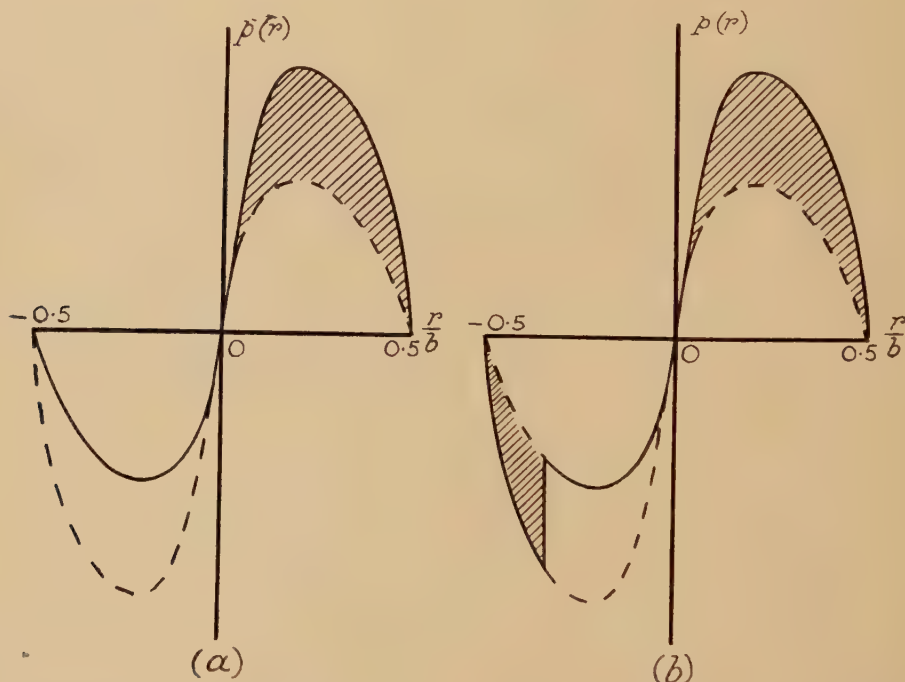
* In some cases, e.g. a micro-crack, the dislocation will be attracted towards the barrier, rather than repelled; and if it can proceed through then $\tau(0)$ will be the minimum.

value at $s=0$, i.e. $\tau_{\max}=\tau(0)$, as can be seen by reference to figs. 8 (a) and 8 (b). In fig. 8 (a) the dislocation is at the barrier, and the shaded area represents $\tau(0)$. In fig. 8 (b) the dislocation is at a distance s beyond the barrier and the discontinuity occurs as shown, i.e. $\tau(s)$ is the sum of the shaded areas (of opposite sign) and therefore less than $\tau(0)$.

§4. ARRAYS OF DISLOCATIONS

We now consider the more general problem of an array of n like dislocations piled up against a barrier by the stress τ (fig. 9). The integral eqn. (14) still applies here, but has not yet been solved. However, the

Fig. 8



The general form of the $p(r)$ function (full line) when the centre of the dislocation lies (a) at the barrier, and (b) inside the barrier.

applied shear stress may be estimated in much the same way as before, the only difference arising from the altered boundary conditions which are now

$$\left. \begin{aligned} \psi(+\infty) &= -(n/2) + (\tau/\mu), \\ \psi(-\infty) &= (n/2) + (\tau/\mu). \end{aligned} \right\} \quad \dots \dots \dots (21)$$

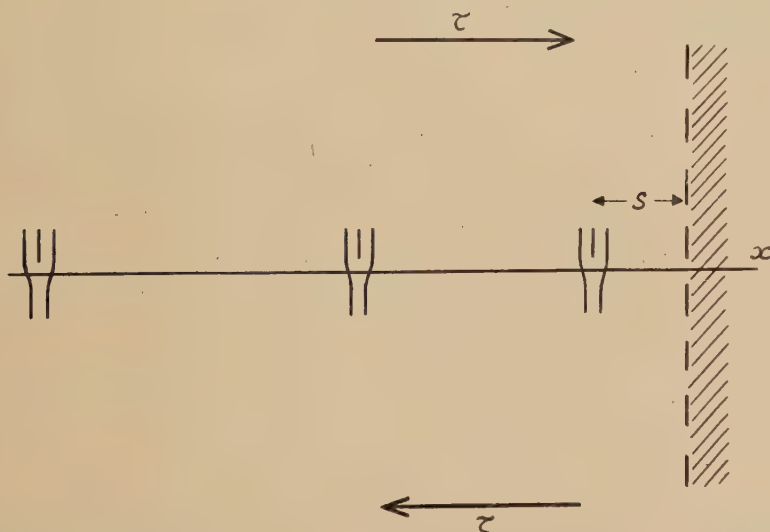
It follows from this that $[\psi(x)]_{-\infty}^{\infty} = -n$ so that the result (15) is modified to

$$\tau = \frac{1}{n} \int_{-1/2}^{1/2} p\left(\frac{r}{b}\right) d\left(\frac{r}{b}\right) = \frac{I}{n}, \quad \dots \dots \dots (22)$$

the integral I having the same geometrical interpretation in the (p, r) diagram as before. It is still necessary to know the relative displacement at $x=s$, but in most cases an estimate of the type (18) may again be used, provided that the dislocations behind the first are relatively widely spaced. This is usually the case if τ is small, because pairs of like dislocations at a distance R apart repel one another as $(1/R)$ (Cottrell 1949) whilst a barrier usually repels dislocations as $(1/s^3)$ or $(1/s^4)$; accordingly s will be small compared with the spacings of the other dislocations.

It will be noted that the integral I in (22) is identical with that occurring in (17),* which gives the external shear stress required to hold a single dislocation at a given distance s from the barrier. It follows therefore that bI (using the Mott-Nabarro relation $F=\tau b$) is the repulsive force exerted by the barrier on a dislocation at a distance s from it, i.e. bI is the repulsive force exerted on the first dislocation of the array above.

Fig. 9



Like edge dislocations piled up against a barrier by an external shear stress τ .

Since this dislocation is in equilibrium, the force exerted on it by the external shear stress τ plus the other $n-l$ dislocations of the array is given by bI , which is equal to $nb\tau$ from (22). This is a well-known result, first derived by Cottrell (1949) from a virtual work calculation.

The equilibrium positions of an array of n Burgers dislocations under an external shear stress τ , relative to the first 'locked' one, have been calculated by Eshelby, Frank and Nabarro (1951) as the n roots of the equation

$$xL_n' \left\{ -\frac{2\pi\tau}{\mu} x \right\} = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

* Provided that the same estimate of $r(s)$ is used in both integrals.

where L_n is the Laguerre polynomial of order n and x is in β -units. These results, together with the preceding calculations, suffice to determine the equilibrium positions taken up by an array of dislocations when their path is blocked by some type of barrier in the lattice, i.e. the position of the first one is determined from (22) while the positions relative to that of the others are given by (23), the interaction between those and the barrier being insignificant.

§5. APPROXIMATE SOLUTION OF THE INTEGRAL EQUATION

In order to find the relative displacement r at any point on the slip direction, and in particular at the discontinuity, it is necessary to solve eqn. (14) for the displacement function ψ . When $r(s)$ is known then $\tau(s)$ can be found from the relation (17) without reference to approximations of the type (18).

The eqn. (14) is very difficult to solve exactly, since it is non-linear and does not fall into any standard form. Many of the inhomogeneities, however, which are met with in practice (apart from Zener micro-cracks) constitute 'weak' barriers, involving only a small change from one law of force to another. This suggests accordingly that we adopt a perturbation technique, regarding the Peierls case as a first approximation, since this has been solved exactly. This procedure leads to a linear integral equation of which a series solution has been found in terms of a certain complete set of functions.

To effect the linearization, we write

$$\left. \begin{aligned} p[\psi] &= p_0[\psi] + p_1[\psi], \\ \psi(x) &= \psi_0(x) + \psi_1(x), \end{aligned} \right\} \dots \dots \dots (24)$$

where

$$\left. \begin{aligned} p_0[\psi] &= -(\mu/2\pi) \sin 2\pi\psi, \\ \psi_0(x) &= -(1/\pi) \tan^{-1} x, \end{aligned} \right\} \dots \dots \dots (25)$$

these constituting the zero order solution and satisfying, from (10), the relation

$$p_0[\psi_0(x)] = \frac{\mu}{\pi} \int_{-\infty}^{\infty} \frac{d\psi_0(t)}{dt} \frac{dt}{t-x} \dots \dots \dots (26)$$

The quantities $p_1[\psi]$ and $\psi_1(x)$, where the perturbation p_1 is of course a given function of ψ , are assumed to be first order, so that we may write

$$\begin{aligned} p[\psi] &= p_0[\psi_0 + \psi_1] + p_1[\psi_0 + \psi_1] \\ &\simeq p_0[\psi_0] + \psi_1 p_0'[\psi_0] + p_1[\psi_0] + \psi_1 p_1'[\psi_0], \end{aligned}$$

on applying Taylor's theorem and retaining only zero and first order terms. The product $\psi_1 p_1'[\psi_0]$ is also neglected as being effectively second order, for, although in general the derivative of a first order quantity is not necessarily first order, p_1' is of the same order as p_1 , as can be seen

on physical grounds by the aid of fig. 6. Accordingly, on substituting $p=p_0+\psi_1 p'_0+p_1$ into (14) and cancelling the zero order terms on each side, we obtain

$$\psi_1(x)p'_0[\psi_0(x)]+p_1[\psi_0(x)]=\frac{\mu}{\pi}\int_{-\infty}^{\infty}\frac{d\psi_1}{dt}\frac{dt}{t-x}+\tau, \quad (27)$$

which is a linear integral equation for the unknown function $\psi_1(x)$. The boundary conditions are

$$\psi_1(\pm\infty)=(\tau/\mu), \quad (28)$$

$$\psi_1(0)=0, \quad (29)$$

the former following from (5) and (24), and the latter from the condition that the centre of the dislocation is to be fixed at the origin. A series solution for this equation with these boundary conditions is derived in § 6.

In cases where the perturbations are not everywhere first order, it is necessary to be able to estimate the magnitude of the errors involved in the above method. On adding eqns. (26) and (27), and comparing with (7), it will be seen that the stress $p(x)$ corresponding to the displacements

$$\psi(x)=\psi_0(x)+\psi_1(x),$$

where $\psi_1(x)$ is given by (27), is simply

$$p(x)=p_0[\psi_0(x)]+p_1[\psi_0(x)]+\psi_1(x)p'_0[\psi_0(x)].$$

On eliminating x between $p(x)$ and $\psi(x)$ we can find the function $p(\psi)$, and hence $p(r)$, for which the above $\psi(x)$ is the *exact* solution. The deviations from the required laws of force in each region of the slip plane provide a measure of the errors involved in the method; a typical example is shown in fig. 11.

§ 6. SERIES SOLUTION

A solution for ψ_1 may be obtained by expanding it as a Fourier series of argument $2 \tan^{-1} x$, so that

$$\psi_1(x)=a_0+\sum_{n=1}^{\infty}\{a_n C_n(x)+b_n S_n(x)\}, \quad (30)$$

where*

$$C_n(x)=\cos (2n \tan^{-1} x); C_n(\pm\infty)=(-1)^n,$$

$$S_n(x)=\sin (2n \tan^{-1} x); S_n(\pm\infty)=0,$$

* These functions do not form an orthonormal set in the range $[-\infty, \infty]$ unless multiplied by the factor

$$\left(\frac{2}{\pi(1+x^2)}\right)^{1/2},$$

as follows from the properties of trigonometrical functions of argument $2 \tan^{-1} x$.

and where a_n and b_n are constants to be determined. The condition (28) is automatically satisfied by re-writing (30) as

$$\psi_1(x) = (\tau/\mu) + \sum_{n=1}^{\infty} \{a_n[C_n(x) - (-1)^n] + b_n S_n(x)\}, \quad (31)$$

since the terms in the summation vanish at infinity.

Three important properties of $C_n(x)$ and $S_n(x)$, which we shall make use of, are as follows:—

(1) $C_n(x) - (-1)^n$ and $S_n(x)$ are conjugate functions in the sense defined by Titchmarsh (1948), i.e. they satisfy the integral relations

$$\left. \begin{aligned} \frac{1}{\pi} \int_{-\infty}^{\infty} \{C_n(t) - (-1)^n\} \frac{dt}{t-x} &= -S_n(x), \\ \frac{1}{\pi} \int_{-\infty}^{\infty} S_n(t) \frac{dt}{t-x} &= C_n(x) - (-1)^n, \end{aligned} \right\} \quad (32)$$

where n is any positive integer (see Appendix I).

(2) The derivative of any function of the set may be expressed as a linear combination of other functions of the set, i.e.

$$\left. \begin{aligned} S_n' &= (n/2)(C_{n-1} + 2C_n + C_{n+1}), \\ C_n' &= -(n/2)(S_{n-1} + 2S_n + S_{n+1}), \end{aligned} \right\}, \quad (33)$$

as may be easily verified.

(3) The product of any two functions may also be expressed in a similar manner, i.e.

$$\left. \begin{aligned} S_n S_m &= \frac{1}{2}(C_{n-m} - C_{n+m}), \\ C_n C_m &= \frac{1}{2}(C_{n+m} + C_{n-m}), \\ S_n C_m &= \frac{1}{2}(S_{n+m} + S_{n-m}), \end{aligned} \right\} \quad (34)$$

from elementary trigonometrical formulae.

In order to expand $p_1[\psi_0]$ in terms of the above functions, we write $p_1[\psi]$ as a Fourier series, so that

$$p_1[\psi] = \mu \sum_{n=0}^{\infty} \{A_n \cos 2n\pi\psi + B_n \sin 2n\pi\psi\}, \quad (35)$$

$$= \mu \sum_{n=0}^{\infty} (-1)^n \{A_n \cos 2n\pi r/b + B_n \sin 2n\pi r/b\}, \quad (36)$$

from (3). We note that the stress is zero when $r=0$, i.e. when the atoms are in alignment, giving

$$\sum_{n=0}^{\infty} (-1)^n A_n = 0, \quad (37)$$

and hence from (25), (35) and (37),

$$p_1[\psi_0] = \mu \sum_{n=1}^{\infty} \{A_n[C_n(x) - (-1)^n] - B_n S_n(x)\}, \quad (38)$$

On substituting into the integral eqn. (27) for $\psi_1(x)$ from (31), $p_1[\psi_0(x)]$ from (38), and $p_0'[\psi_0(x)]$ from (25), it is readily seen by virtue of the properties (32–34) that all terms in (27) can be expressed as linear

combinations of the functions $C_n(x) - (-1)^n$ and $S_n(x)$. On comparing their respective coefficients on both sides of the equation, we are led to the following set of relations:—

$$\left. \begin{aligned} -2(\tau/\mu) + a_2 + 2 \sum_{n=2}^{\infty} a_n (-1)^n &= -2A_1, \\ na_{n+1} + 2na_n + (n-2)a_{n-1} &= -2A_n, \quad (n \geq 2), \end{aligned} \right\} \quad \cdot \quad \cdot \quad (39)$$

$$\left. \begin{aligned} 2b_1 + b_2 &= 2B_1, \\ nb_{n+1} + 2nb_n + (n-2)b_{n-1} &= 2B_n, \quad (n \geq 2). \end{aligned} \right\} \quad \cdot \quad \cdot \quad \cdot \quad (40)$$

These include all the coefficients a_n and b_n except a_1 , which is found from (29) and (31).

It will be seen that a_n and b_n are specified by two quite independent sets of relations, but except for $n=1$ these are both second order difference equations of the type

$$ny_{n+1} + 2ny_n + (n-2)y_{n-1} = X_n \quad (n \geq 2). \quad \cdot \quad \cdot \quad \cdot \quad (41)$$

Since y_1 does not appear in the relation for $n=2$, the general solution contains only one arbitrary constant; this can be found from the condition that y_n must tend to zero as $n \rightarrow \infty$, as otherwise the series solution (21) is divergent. It may be verified (Appendix II) that the required solution of (41) is

$$y_n = -Y_n \sum_{s=n}^{\infty} \frac{1}{(s-1)sY_s Y_{s+1}} \sum_{t=2}^s (t-1)X_t Y_t, \quad (n \geq 2), \quad \cdot \quad \cdot \quad (42)$$

where Y_n is the complementary function, i.e. the solution of (41) when $X_n=0$. A standard method (Boole 1946) yields the result

$$Y_n = \frac{1}{2}(-1)^n \left\{ \frac{L_{n-1}(-2)}{(n-1)!} - \frac{L_{n-2}(-2)}{(n-2)!} \right\}, \quad (n \geq 2), \quad \cdot \quad \cdot \quad (43)$$

where $L_n(x)$ is the Laguerre polynomial of order n , as may be directly verified by substitution.

The external shear stress τ can be calculated directly from the relations (39) in the following manner. Multiplying through each relation by $(-1)^n$, including the first, summing from $n=1$ to $n=\infty$, and making use of the identity

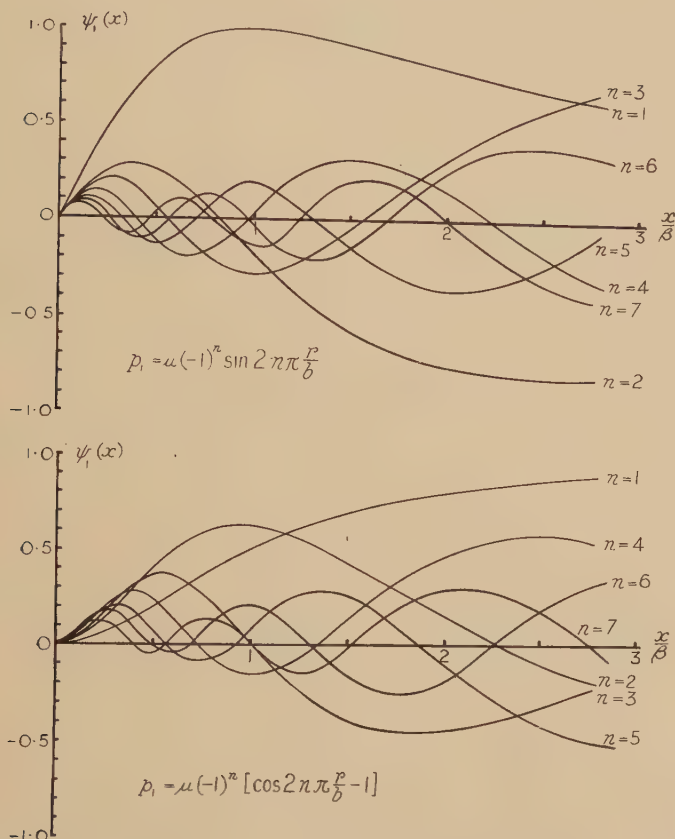
$$\sum_{n=1}^{\infty} n(-1)^n a_{n+1} + \sum_{n=2}^{\infty} (n-2)(-1)^n a_{n-1} \equiv -2 \sum_{n=2}^{\infty} (n-1)(-1)^n a_n$$

leads to the result

$$\begin{aligned} \tau &= -\mu \sum_{n=1}^{\infty} (-1)^n A_n \\ &= \mu A_0 \quad \text{from (37)} \\ &= \int_{-1/2}^{1/2} p_1 \left(\frac{r}{\bar{b}} \right) d \left(\frac{r}{\bar{b}} \right) \quad \text{from (36).} \end{aligned}$$

is now simply obtained by taking the appropriate linear combination of the separate harmonic ψ_1 's. A typical displacement curve, which was obtained in this way, is shown in fig. 11.

Fig. 10



Graphs of $\psi_1(x)$ for harmonic perturbations to the Peierls law of force.

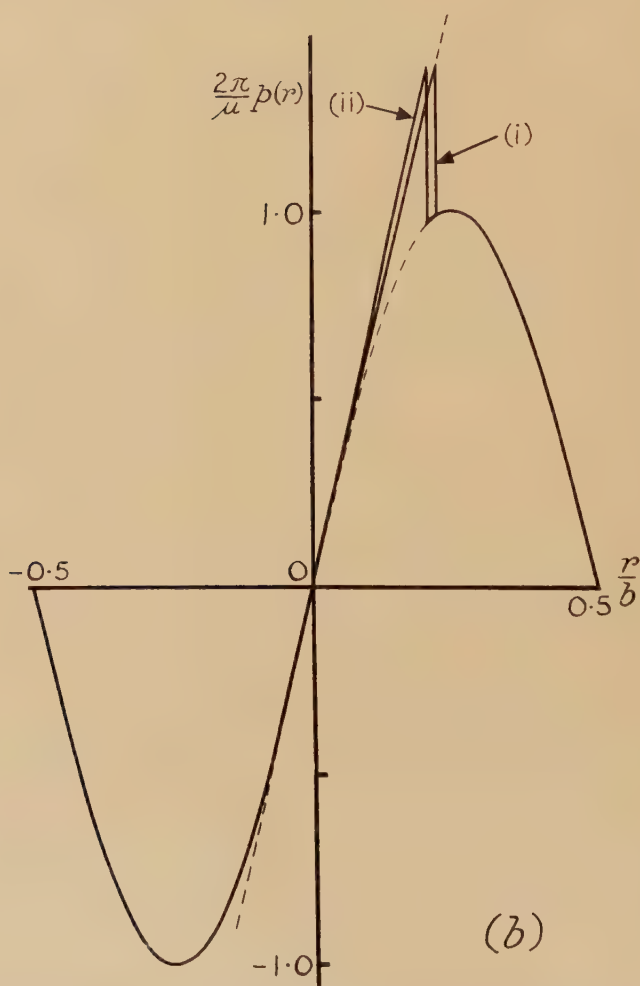
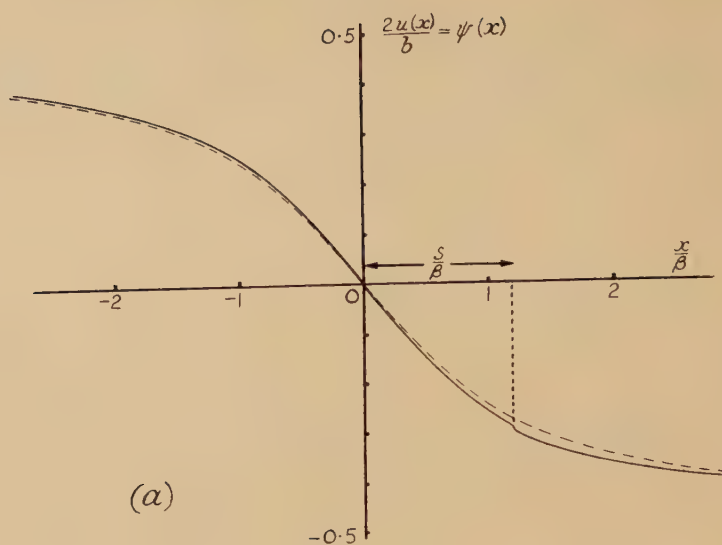
APPENDIX I

The pair of integral relations (32) may be most conveniently derived by considering the contour integral

$$\oint \left\{ \left(\frac{1+iz'}{1-iz'} \right)^n - (-1)^n \right\} \frac{dz'}{z'-x},$$

where the path of integration is the real axis from $-R$ to R ($R > x$) together with a semi-circle of radius R in the upper half-plane. The

Fig. 11



(a) Displacement curve for a dislocation at one lattice spacing from a Hooke's law barrier (the broken curve is the zero order solution). (b) The law of force. Curve (i) is the function $p(r)$ for which the perturbation method yields the approximate solution (a), whilst (ii) is the function which corresponds exactly to (a). The latter differs from Hooke's law by not more than 5%, and with negligible errors in the sinusoidal region.

integrand has a simple pole on this contour at $z'=x$ (at which the Cauchy principal value is taken), so that from the calculus of residues

$$\oint \left\{ \left(\frac{1+iz'}{1-iz'} \right)^n - (-1)^n \right\} \frac{dz'}{z'-x} = \pi i \left\{ \left(\frac{1+ix}{1-ix} \right)^n - (-1)^n \right\}.$$

The semi-circular contour C gives no contribution to the integral in the limit when $R \rightarrow \infty$, since

$$\left| \int_C \right| \leq \left| \left(\frac{1+iz'}{1-iz'} \right)^n - (-1)^n \right|_{\max} \frac{\pi R}{|R-|x||} \rightarrow 0 \text{ as } R \rightarrow \infty.$$

Furthermore, since $\tan^{-1}x = (1/2i) \log (1+ix)/(1-ix)$, we have

$$\left(\frac{1+ix}{1-ix} \right)^n = e^{2ni \tan^{-1} x} = C_n(x) + iS_n(x).$$

Thus on separating out real and imaginary parts, it follows that

$$\begin{aligned} \frac{1}{\pi} \int_{-\infty}^{\infty} \{C_n(x') - (-1)^n\} \frac{dx'}{x'-x} &= -S_n(x), \\ \frac{1}{\pi} \int_{-\infty}^{\infty} S_n(x') \frac{dx'}{x'-x} &= C_n(x) - (-1)^n, \end{aligned}$$

which are the integral relations required.

APPENDIX II

We write $(n-1)y_n = z_n$ and $(n-1)Y_n = Z_n$, where y_n is the complete solution and Y_n is the complementary function (43), so that from (41)

$$\left. \begin{aligned} z_{n+1} + \frac{2n}{n-1} z_n + z_{n-1} &= X_n, \\ Z_{n+1} + \frac{2n}{n-1} Z_n + Z_{n-1} &= 0. \end{aligned} \right\} \quad n \geq 2.$$

Multiplying the first of these equations by Z_n and the second by z_n , and subtracting gives

$$\Delta \{z_n Z_{n-1} - z_{n-1} Z_n\} = X_n Z_n,$$

and hence

$$\Delta \left\{ Z_{n-1} Z_n \Delta \left(\frac{z_{n-1}}{Z_{n-1}} \right) \right\} = X_n Z_n,$$

where Δ is the finite difference operator ($\Delta f_n = f_{n+1} - f_n$). This integrates to

$$Z_{n-1} Z_n \Delta \left(\frac{z_{n-1}}{Z_{n-1}} \right) = \sum_{t=1}^{n-1} X_t Z_t + A,$$

where A is a constant which is found to be zero on setting $n=2 (z_1=Z_1=0)$, and further to

$$\frac{z_{n-1}}{Z_{n-1}} = - \sum_{s=n}^{\infty} \frac{1}{Z_{s-1} Z_s} \sum_{t=1}^{s-1} X_t Z_t + B.$$

From this it follows, on substituting for z_n and Z_n and stepping up s and n by unity, that

$$y_n = -Y_n \sum_{s=n}^{\infty} \frac{1}{(s-1)sY_sY_{s+1}} \sum_{t=2}^s (t-1)Y_tX_t + BY_n.$$

This contains one arbitrary constant and is therefore the general solution of (41). From (43) we find that the function Y_n oscillates and diverges for large values of n , so that we choose $B=0$, giving the solution (42).

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XVII. *The Application of Proportional γ -ray Counters to the Determination of the Decay Scheme of I^{131}*

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ABSTRACT

A naphthalene-anthracene crystal has been used to separate the high energy γ -rays in I^{131} from the main low energy component. Coincidence absorption measurements with β -rays and the high energy γ -rays show that the latter are in cascade with β -rays of maximum energy 315 kev. Consideration of the bias curve of these coincidences however shows that the coincidence β -spectrum is complex and may be analysed into components of maximum energy 328 and 246 kev in the ratio (2 : 1) expected if they are in cascade with the 638 and 720 kev γ -rays respectively, which are known to be emitted. This suggests that the latter are emitted in transitions to the ground state of Xe^{131} .

§1. INTRODUCTION

IN recent years I^{131} has been investigated by many workers, but, although there is a large measure of agreement as to the energy of the main β -ray, and the number and energy of the various γ -rays, it has so far proved impossible to fit them together into a completely self-consistent decay scheme which is generally acceptable. The present paper describes coincidence measurements made using proportional scintillation γ -ray counters, which it is believed resolve many of the difficulties associated with the establishment of such a scheme.

Briefly recapitulating results which are generally accepted, there is a main β -ray of energy very close to 600 kev, and there are γ -rays of energy 363, 283, 80 and 640 kev, (Metzger and Deutsch 1948 and others). There is an isomeric transition of energy 163 kev with a half-period of 12 days (Brosi, Dewitt and Zeldes 1949). More recently there have been reported low intensity γ -rays of energy 177 kev, (Cork *et al.* 1951) and 720 kev (Zeldes, Brosi and Ketelle 1951). The latter has also been observed in both the primary and secondary electron spectra in this laboratory,* and is of about half the intensity of the 640 kev γ -ray. In addition the last-mentioned workers have reported the existence of a 1% branch of a β -ray of maximum energy 810 kev, also observed in this

† Communicated by the Author.

* This, with other data which will be quoted in this paper, and also marked with an asterisk, is to be made the subject of a further paper.

laboratory, and possibly associated with the isomeric level in Xe^{131} . As for intensities, data on which have recently been tabulated by Feister and Curtiss (1950), the 363 kev γ -ray is by far the most intense, the 283 and 80 kev are estimated to be emitted in about 5% of the transitions, while the 640 and 720 kev γ -rays together are emitted in about 15% of the disintegrations. The other γ -rays are each of about 1% intensity.

As a result of the coincidence measurements of Metzger and Deutsch (1949) and the high precision measurement of the energies of the 363, 283 and 80 kev γ -rays by Lind *et al.* (1949), it can now be taken as established that the 283 and 80 kev γ -rays in cascade form a parallel branch to the 363 kev γ -ray. As was to be expected the most intense γ -ray is in cascade with the most intense β -ray. In addition Metzger and Deutsch showed that no appreciable fraction of the 80 kev γ -rays could be in coincidence with β -rays of energy much less than the 600 kev β -ray, and this rules out the possibility that the 80 kev γ -ray could be in one to one coincidence with one of the higher energy γ -rays. That part of the decay scheme about which there is no dispute is indicated in fig. 5. The chief point of controversy is the measurement of the energy of a low energy β -group of about 15% intensity, which according to Kern, Mitchell and Zaffarano (1949) is 250 kev and according to Metzger and Deutsch (1949) is 315 kev, all measurements being made on the basis of deviations from the straight line Fermi plot of the primary spectrum. We have already briefly reported the result of a measurement using a proportional scintillation γ -ray counter in coincidence with a fast β -counter, which is in agreement with that of Deutsch, Cavanagh (1950). We will first describe this work more fully.

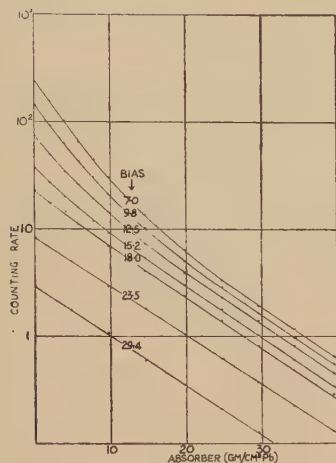
§ 2. THE MEASUREMENT OF THE ENERGY OF THE β -RAYS IN COINCIDENCE WITH THE HIGH ENERGY γ -RAYS OF I^{131}

The use of organic crystal scintillators as a crude spectrometer for γ -rays was originally reported by Jordan and Bell (1949) and this work was done shortly afterwards. The crystal used, of naphthalene with a small percentage of anthracene, took the form of a disc 2 in. in diameter and $\frac{1}{2}$ in. thick, which was sealed onto the end window of an E.M.I. multiplier type VX. 5032. This was followed by a pre-amplifier and a 2 Mc/s amplifier. The pulses were analysed by means of a simple discriminator. Measurements made in this way of the pulse distribution due to the γ -rays of various isotopes such as Na^{24} , Au^{198} , and I^{131} did in fact show structure due to the complexity of the γ -ray spectra in very much the same way as that reported by Jordan and Bell.

The object of the present experiment with I^{131} was to choose a value of bias such that none of the main group of 363 kev γ -rays was recorded, so that coincidence measurements with β -rays and the high energy γ -rays alone might be made. In order to determine the point on the bias curve at which all the lower energy γ -rays were eliminated, a series of lead absorption measurements in a well-collimated geometry, was made at various values of the bias. This set is shown in fig. 1. It is evident

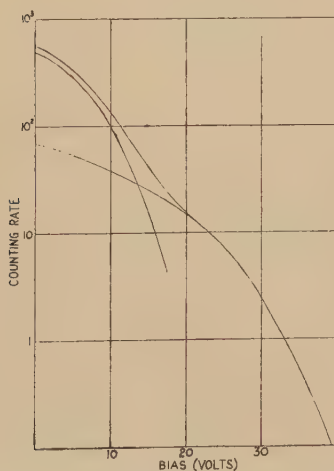
that at low values of the bias the recorded absorption curve is complex, and that as the bias is increased the soft component diminishes with respect to the hard component, until at 20 volts bias it has completely disappeared. The two components appear to have exponential absorption curves with coefficients close to those expected for the 363, 640 and

Fig. 1



Lead absorption curves for various values of discriminator bias for γ -rays of I^{131} .

Fig. 2



Bias curves for the main γ -rays of I^{131} .

720 kev γ -rays. So it is possible to extrapolate the curve for the hard component to zero absorber thickness, and thus determine the relative proportion of the two components present at any value of bias. This makes it possible to analyse the complex bias curve into the bias curves due to the two components and this is done in fig. 2. To obtain some

idea of the efficiency of the crystal for counting these γ -rays, it was replaced by a copper-walled geiger counter, which gave a counting rate of 20 per sec as against 5 000 per sec for the crystal. The ratio of 250/1 fits in well with the calculated efficiency of the crystal for 360 kev γ -rays of about 50%. That for the high energy component is about 35%, so even after the effect of the 360 kev γ -ray has been completely suppressed, the higher energy γ -ray is still counted with an intrinsic efficiency of 5%.

§3. β - γ COINCIDENCE MEASUREMENTS

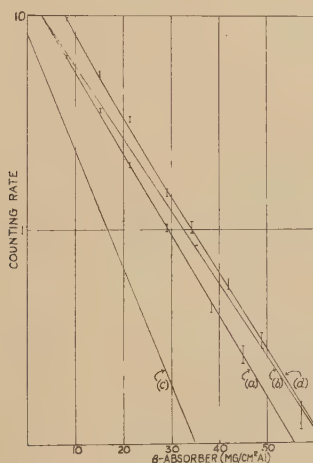
In the coincidence measurements the response from the crystal was biased off just beyond the end point of the 360 kev component. The β -rays were recorded by means of an end window geiger counter operated at about 50 volts below the threshold, in the semi-proportional region, so as to eliminate dead-time, and working into a 2 Mc/s amplifier. We have used such counters at rates up to 50 000 per sec. Both amplifiers were followed by discriminators, feeding in turn into a conventional coincidence unit. All rates were recorded simultaneously with type 200 scalars.

An effectively weightless source was deposited on mica and sealed in with $\frac{1}{2}$ mg/cm² nylon sheet, and its intensity was such that the genuine coincidence rate with no absorber was about twice the accidental rate. Characteristics of the genuine coincidence rate versus delay, and also coincidence resolving time, were taken, and such a value of the latter, 0.2 μ sec, was chosen, that not only were all genuine coincidences recorded to within 1%, but some margin was left for small changes in delay caused by variable settings of discriminators, etc. The coincidence resolving time was measured at intervals throughout the experiment at the single rates actually used, and was found to remain constant to within the measured accuracy of 1%.

The source was placed almost in contact with the crystal and a short distance away from the β -counter, so as to permit of the introduction of aluminium absorbers. Coincidences were measured as function of thickness of absorber placed before the β -counter, at least 4 000 coincidences being recorded at each point. The genuine coincidence rate decreased by a factor of over a hundred over the range of absorber thickness used, being twice the accidental rate at zero thickness and falling to about one-third of the accidental rate at the maximum thickness of absorber used. Corrections were made for the cosmic coincidence rate and also for γ - γ and scattered coincidences. Even at the largest thickness of absorber used these corrections were small. The absorption curve of the coincident β -rays was derived in the following way: at each absorber value the ratio genuine/accidental coincidence rate was calculated, with the genuine β - γ rate corrected as described above. The source was replaced by a much smaller one, and the absorption curve of the primary β -rays measured in the same geometry as the coincidence experiment. Each value of the genuine/accidental rate was then multiplied by the counting

rate obtained for the same thickness of absorber in the subsidiary experiment, corrected for backgrounds. This gives a quantity proportional to the detection efficiency of the β -rays in coincidence with the high energy γ -rays at each value of absorber. The reason for this procedure is that it is necessary to eliminate a change in intrinsic efficiency with counting rate in the semi-proportional β -counter, which amounted to some 20% over the range of counting rates used. The absorption curve derived in this way is shown in fig. 3. It is essentially exponential over the range of measurement. A comparison was made with the absorption curve of Co^{60} β -rays, measured in the same geometry, under the same back-scattering conditions, and at small counting rates. It is interesting to note that the estimated errors in this measurement are comparable with those of the coincidence measurement, arising in this case from the large and somewhat uncertain value of the γ -ray background.

Fig. 3



- (a) Absorption curve for Co^{60} β -rays (310 keV).
 (b) and (c) Absorption curves for 328 and 246 keV β -rays derived from (a).
 (d) Experimental points from β - γ coincidences for high energy γ -rays of I^{131} together with composite absorption curve of 328 and 246 keV β -rays.

Inspection shows that the energies of the two β -rays are very nearly equal, and a Feather analysis gives the value for the maximum energy of the β -rays in coincidence with the high energy γ -rays as 315 keV, if the maximum energy of Co^{60} β -rays is taken as 310 keV. This is in agreement with the result of Metzger and Deutsch (1949) and the estimated error of $\sim 5\%$ is such that it is believed to be incompatible with the value of 250 keV derived by other workers. On the basis that the 603 keV β -ray and the 363 keV γ -ray are in cascade, the total energy of the transition is 966 keV; the energies of the two high energy γ -rays as measured in this laboratory* are 638 and 720 keV. The coincidence measurements of Deutsch require that the 80 keV γ -ray is not directly

in cascade with a higher energy γ -ray, hence it is very probable that these two γ -rays are emitted in transitions from levels at 638 and 720 kev above the ground state of Xe^{131} and are associated with β -rays of maximum energy 328 and 246 kev respectively. An energy of 328 kev is undoubtedly compatible with the measured energy of 315 kev, but the 720 kev γ -ray is certainly in coincidence with a β -ray (the small conversion coefficient of 0.27% ensures that it does not belong to an isomeric transition) and this β -ray must be of energy 246 kev, or less (in the case where a further γ -ray is in cascade with the 720 kev γ -ray). As will be shown below, by consideration of the bias curve of β -high energy γ -ray coincidences, it must in fact be in cascade with a β -ray of energy not very much less than 250 kev. The low energy β -group should then consist of two β -spectra of energy 328 and 246 kev in proportion to the relative intensities of the 638 and 720 kev γ -rays, with a correction made for the difference in detection efficiency for these two γ -rays. Absorption curves for the two β -spectra were constructed from the experimental one for Co^{60} β -rays, using the Feather rule, and taking the maximum ranges of 246, 310 and 328 kev β -rays from interpolations in the table given by Gupta and Ghosh (1946). An estimate of the relative detection efficiencies of the 638 and 720 kev γ -rays was made by first comparing the bias curves for the soft and hard components, and from this, estimating the relative bias curves for the two hard components. The relative intrinsic efficiency of the crystal for the latter (at zero bias) was calculated on the basis of Compton absorption. This together with the spectrometer measurement of a relative intensity of 2 : 1 gives the ratio of the mixture of the 328 and 246 kev β -spectra as 1.6 : 1 at the value of bias used for the coincidence measurement. These two absorption curves, together with the composite, constructed on the foregoing basis, are also reproduced in fig. 3. The energy derived from the latter is 310 kev, in excellent agreement with the experimental value.

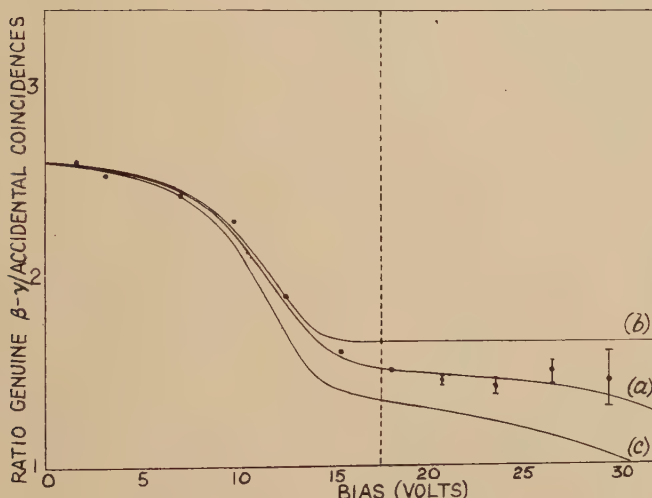
§ 4. BIAS CURVE OF β - γ COINCIDENCES

Further information may be derived from an examination of the bias curve of β - γ coincidences, in this instance with no absorber inserted before the β -counter but only the zero value of 9.0 mg/cm² aluminium equivalent due to the counter window, the source covering, and the air between source and β -counter. If we consider the ratio of the genuine coincidence rate to the product of the single counting rates, then at low values of bias the great majority of the coincidences will be due to 363 kev γ -rays and 603 kev β -rays, the latter not being greatly attenuated in the zero thickness of absorber. At high values of bias however, coincidences between high energy γ -rays and low energy β -rays will become increasingly predominant. The latter are heavily attenuated in the zero absorber thickness, and hence the recorded γ -rays will produce relatively fewer coincidences. Hence one would expect the ratio of genuine coincidences to the product of the single rates, as the bias is

increased from small values, to fall slowly over the region where coincidences due to the main group of low energy γ -rays are predominant ; to fall rapidly in the intermediate region, and to level off beyond the point where the coincidences due to the high energy γ -rays become important.

Such a characteristic was in fact obtained and is shown in fig. 4, all quantities having been corrected for backgrounds. It becomes important to know if this measured characteristic is in fact compatible with the mixture of low energy β -spectra required to be in coincidence with the 638 and 720 kev γ -rays.

Fig. 4



- (a) Experimental points and composite bias curve for 640 and 720 kev γ -rays in coincidence with 328 and 246 kev β -rays respectively.
- (b) Calculated bias curve for γ -ray of 670 kev in cascade with β -ray of 315 kev.
- (c) Calculated bias curve as (a) except low energy β -ray 210 kev.

At this point it becomes necessary to consider an aspect of the bias curves due to γ -rays of different energies which we have only touched on very briefly before. If we consider those values of bias for which the initial counting rate falls by successive factors of 2, then the ratio of corresponding bias values for γ -rays of different energies is constant over a range of counting rates of at least one hundred, and is approximately equal to the ratio of the γ -ray energies. This property is unexpected because the resolution of the crystal would certainly be expected to depend on the energy at least as far as electrons are concerned. However the absorption characteristics of γ -rays, including secondary processes, certainly enter into the question, and make it extremely difficult to handle from the theoretical view point. We have therefore been content to take

this as an empirical property, which has proved extremely useful in separating the components of a complex spectrum, and estimating γ -ray energies.

We may illustrate this property by reference to the bias curve of γ -rays in the experiment under consideration. The relative counting rates of the two components is taken to be that determined in the original absorption measurement; the relevant bias values are given in the following table.

Table 1. Values of Bias Required to Reduce Initial Counting Rate by Successive Factors of Two

$(\frac{1}{2})^n n =$	1	2	3	4	5	6	
Bias value for low energy component	4.1	6.8	8.8	10.0	11.1	12.0	
Bias value for higher energy component	8.2	13.7	16.8	19.2	21.6	23.5	
Ratio high/low	2.00	2.00	1.91	1.92	1.95	1.96	average=1.96

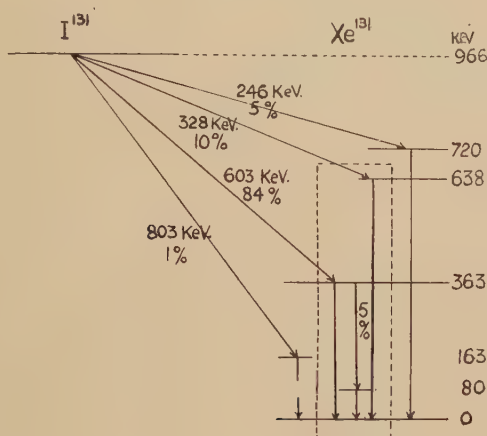
The bias curve of the high energy component should correspond to a mean energy of 670 kev, and therefore its ratio to the energy of the 363 kev γ -ray is 1.85, which compares with the 1.95 derived from the ratio of bias values. For our present purposes however, it is sufficient to take it as strictly proportional and to construct, on this basis, bias curves for the two high energy components separately. We may also from fig. 3 read off the relative efficiencies of detection of the 328 and 246 kev β -rays for 9.0 mg/cm² aluminium absorber, and from an experimental curve determine the attenuation of the main 603 kev component in the same absorber. The ratio of the intensities of the two high energy components is taken, as previously, to be 2:1. All this information enables us to construct a curve (a) corresponding to the experimental points of fig. 4; and the fit is extremely good. On the other hand, a similar bias curve (b) calculated on the assumption of a single γ -ray of energy 670 kev in coincidence with a β -ray of 315 kev is not compatible with the experimental points, the β -rays being attenuated faster in the first 9.0 mg/cm² of absorber than is appropriate to an energy of 315 kev. Taken in conjunction with the previous absorption measurements this implies that the low energy β -spectrum is in fact complex, and it may be resolved into components of energy 328 and 246 kev in the proportion expected if they are in coincidence respectively with the 638 and 720 kev γ -rays. If the lower energy component is assumed to have an energy

as little different as 210 kev, curve (c) in fig. 4 results, which again is inconsistent with the experimental results. This strongly suggests that the 720 kev as well as the 640 kev γ -rays correspond to ground state transitions.

The decay scheme shown in fig. 5 also includes this new evidence. If it is assumed that the known high energy β -spectrum leads to the 163 kev isomeric level, the only known γ -ray not included in the scheme is the 177 kev γ -ray.

These results are not in agreement with the published decay scheme of Bell *et al.* (1951) but later work by the latter group is essentially in agreement with us (private communication).

Fig. 5



Decay scheme for I^{131} . The part enclosed within the broken lines is already fairly well agreed.

§ 5. CONCLUSIONS

β - γ coincidence absorption measurements have shown that the high energy γ -rays in I^{131} are in coincidence with β -rays of maximum energy 315 kev. A consideration of the bias curve of these coincidences, however, shows that the β -spectrum is complex and may be analysed into components of maximum energy 328 and 246 kev in the ratio (2 : 1), expected if they are in cascade with the 638 and 720 kev γ -rays respectively. This suggests that the latter are emitted in transitions to the ground state of Xe^{131} . These results are compatible with the latest measurements of P. R. Bell using a sodium-iodide crystal spectrometer (private communication).

ACKNOWLEDGMENTS

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XVIII. *The Interaction of Fast π -Mesons with Nuclei*

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ABSTRACT

A study has been made of the nuclear disintegrations produced by π -mesons of energy between 50 and 1100 mev, and they have been compared with those produced by protons of energy up to 800 mev. The results are consistent with the hypothesis that a π -meson commonly interacts with a complex nucleus by making two or more elastic collisions with its nucleons. By studying the characteristics of the meson 'stars', estimates have been obtained of the relative probabilities of the meson being absorbed in a nuclear collision, of undergoing charge exchange, or of being elastically scattered.

§1. INTRODUCTION

THE application of the photographic emulsion technique to cosmic-ray studies and the construction of the great synchrocyclotrons, have made possible detailed investigations of nuclear reactions at energies above the threshold for meson production. Considerable progress has thus been made in our knowledge of high-energy nuclear processes, and new problems have emerged, of which one of the most important is the following:

When a nucleon with an energy above 1 bev collides with a nucleus, a large fraction of the available energy is absorbed in the creation of π -mesons. These mesons interact strongly with the nucleons, so that they may be involved in secondary interactions within the parent nucleus and contribute to its disintegration. How important are such interactions and what are their detailed characteristics?

This problem may be approached in a number of ways, one of which is to study the disintegrations produced by π -mesons which have escaped from the parent nucleus. Such studies have been made by Bernardini *et al.* (1950, 1951) using artificially generated π -mesons of energy less than 110 mev. Although similar experiments using π -mesons in the cosmic radiation are more arduous, they allow the study to be extended to particles of much greater energy and this paper describes preliminary results in the interval up to 1.1 bev. In several instances the statistical weight of the observations is small, but the analysis has been made in order to indicate the method of approach which may be adopted.

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§2. IDENTIFICATION AND MEASUREMENTS ON THE STARS PRODUCED BY MESONS AND PROTONS

In the matter of photographic plates exposed to the cosmic radiation, nuclear explosions are produced which are accompanied by the creation of π -mesons. Many of these particles escape from the assembly, but some make secondary nuclear disintegrations within it. The problem of recognizing these particular events can be solved in one of two ways :

It sometimes happens that a nuclear collision involving the production of many 'shower particles' occurs in the emulsion, and that one of these 'shower particles' makes a second collision in the same plate. These events are referred to as 'double stars'. It is possible to identify the particle linking the two disintegrations as a π -meson if the track is sufficiently long and if the mean angle of multiple scattering, $\bar{\alpha}$, is greater than 0.0226° per 100μ . In such events the direction of motion of the π -meson is generally unambiguous—the probability is very small that the particle assumed to produce the second disintegration really emerged from it—but they are rare. Thus, in an examination of 125 cm^3 of emulsion, the observers in this laboratory have recorded 30 000 stars, from which a total of 14 800 shower particles were ejected. Of these, only 40 were observed to make a nuclear interaction before leaving the emulsion, and only 15 of them were identified as π -mesons by scattering and grain density measurements. These numbers are so small that we cannot expect to obtain observations of sufficient statistical weight by this method alone. A second method was therefore employed.

In the second method, each isolated star recorded by the observers was examined for associated tracks of minimum ionization. Scattering and grain-density measurements were made on those tracks which were (a) in the 'upper hemisphere', and (b) of sufficient length in the emulsion to allow $\bar{\alpha}$ to be determined. From the tracks thus selected, only those with a mean angle of scattering, $\bar{\alpha}$, $\geq 0.0226^\circ$ per 100μ were accepted for further study, i.e. only those which gave sufficient evidence to allow the nature and energy of the primary particle to be determined. (Fowler 1950.) A proton or π -meson so identified was regarded as having produced the star with which it was associated if it possessed sufficient energy to cause the disintegration.*

Results of the scattering and grain-density measurements, for these 'primary' particles and for those identified from the double stars, are represented in fig. 1. The resolution between protons and mesons is satisfactory. The identification was ambiguous in only two cases, and they were excluded from the observations. It will be convenient to refer to the 'stars' produced by the two types of particle as 'meson disintegrations' and 'proton disintegrations' respectively.

The value of the scattering parameter $\bar{\alpha} = 0.0226^\circ$ per 100μ corresponds to a kinetic energy of 800 mev for a proton, and 1100 mev for a π -meson. The investigation was therefore confined to particles with energy less than

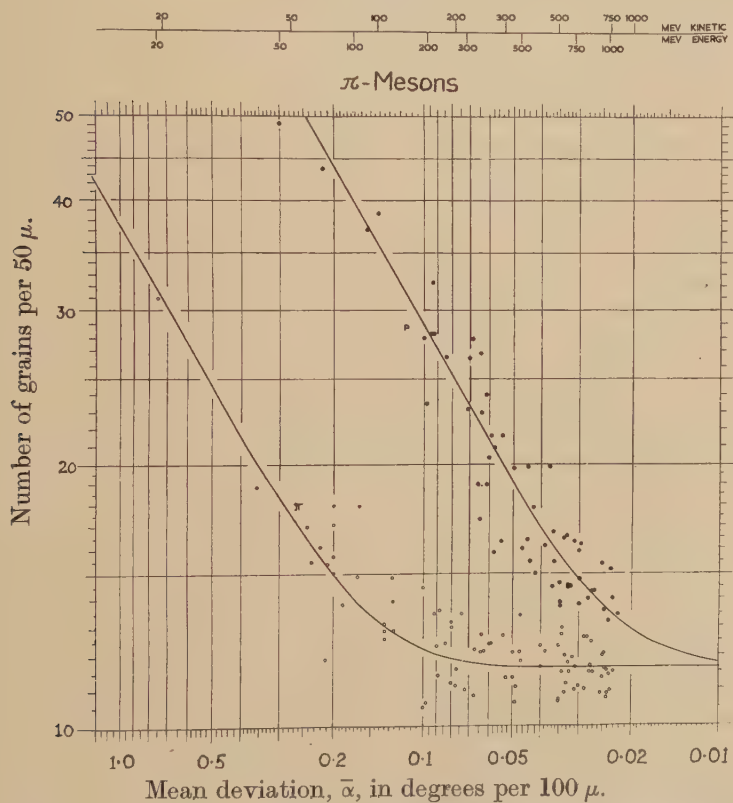
* Since none of the emulsions searched were exposed under dense materials, the number of stars produced by μ -mesons is very small.

these values. The numbers of disintegrations available for analysis, produced by protons and mesons, were 70 and 86, respectively.

The second of the methods described above, will lead, in a small proportion of the events, to errors of identification, and some of the 'primary' particles will, in fact, be secondary particles which originate in stars produced by fast neutrons. In such events there is a certain probability of a shower particle being ejected 'backwards'; and, although rarely, it may satisfy the criteria which lead us to regard it as a 'primary' particle. The proportion of such cases of wrong identification has been estimated by Camerini *et al.* (1951, VI), and shown to be less than 10%.

Fig. 1

Protons



Distribution in the values of $\bar{\alpha}^0$ (100 μ) and grain density of mesons and protons identified as the primary particles of nuclear disintegrations.

This second method also suffers from the disadvantage that the observers only record stars with $N_h \geq 3$, and they miss a substantial fraction of the stars with $N_h = 3$ and 4 (Rochester and Rosser 1951). Some of the smaller stars produced by π -mesons therefore escape observation. The disintegrations observed by the first method are, however, recorded with an efficiency which is independent of N_h . A comparison between the results of the two methods indicates that about 70% of the total number of meson stars

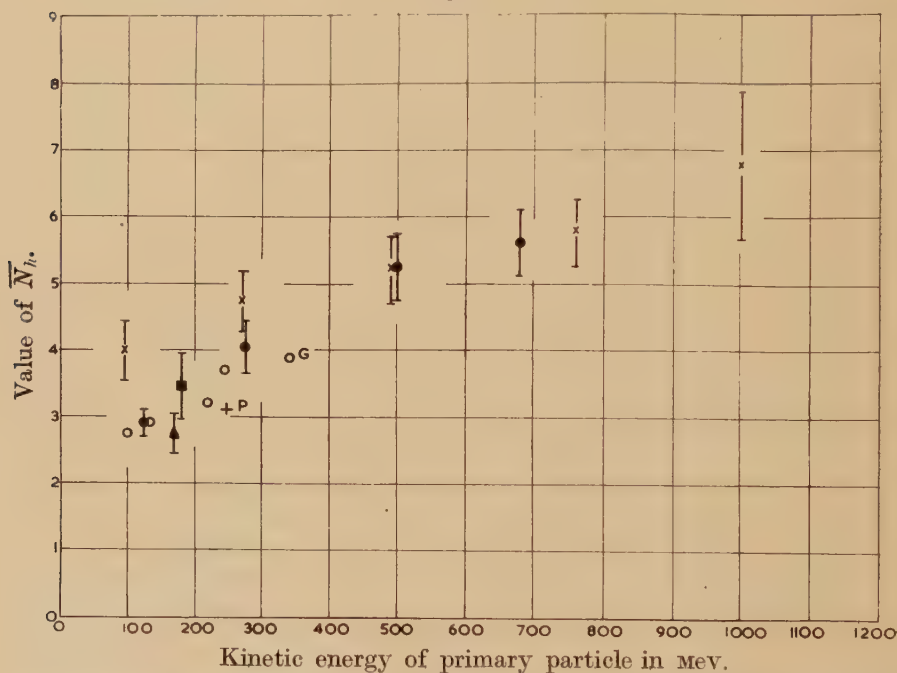
occurring in our emulsions have been observed, the missing 30% corresponding to about half the stars with $N_h \leq 4$.

Measurements on 'Stars' with Identified Primary Particles

The stars produced by protons of energy less than 800 mev or mesons of less than 1100 mev, were analysed in the following manner. The length of each track associated with the star, its grain-density and angle in space with respect to the primary particle were measured. In favourable cases, scattering measurements were also made on the secondary particles. In addition, any particles which reached the end of their range in the emulsion were identified by inspection.

From these observations, the variation of a number of parameters characteristic of the disintegrations could be determined as a function of the primary energy, for stars produced by both mesons and protons. Fig. 2

Fig. 2



Variation with primary energy of the average number of heavily ionizing tracks, $\bar{N}_h$, from meson ($\times$) and proton ($\bullet$) disintegrations.

■ Data for mesons from double stars.

▲ Data for protons from double stars.

oG Results of Germain (1951) for artificially accelerated protons.

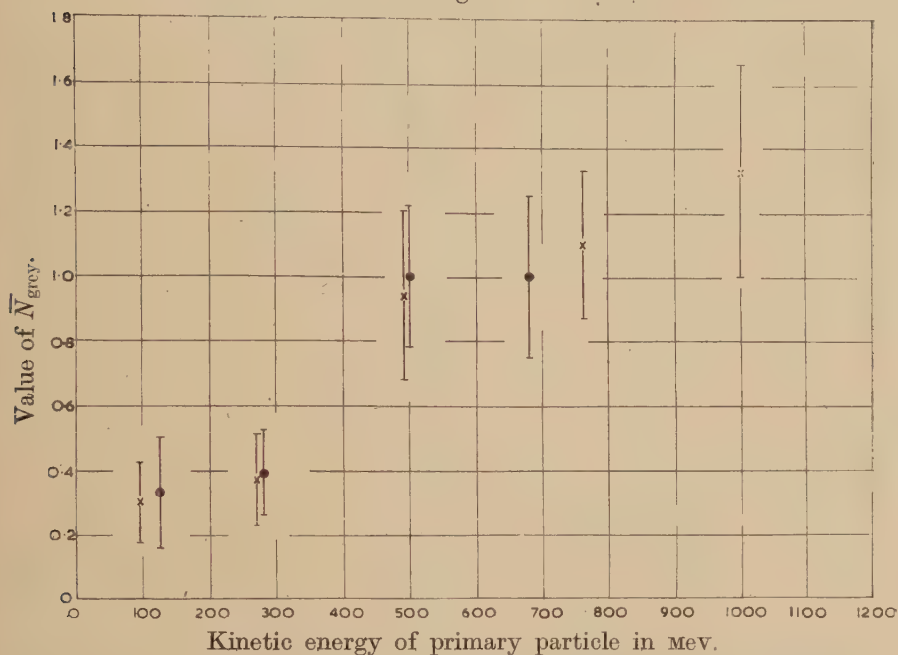
+P Result of Perry (1951) for artificially accelerated protons.

shows the variation of the average number of heavily ionizing branches $\bar{N}_h$; fig. 3 the average number of 'grey' tracks $\bar{N}_{\text{grey}}$; and fig. 4 the average energy of the 'grey' particles, $\bar{E}_{\text{grey}}$.

Values of the mean-free-path for interaction of both π -mesons and protons were obtained from an analysis of the 'double stars', the calcula-

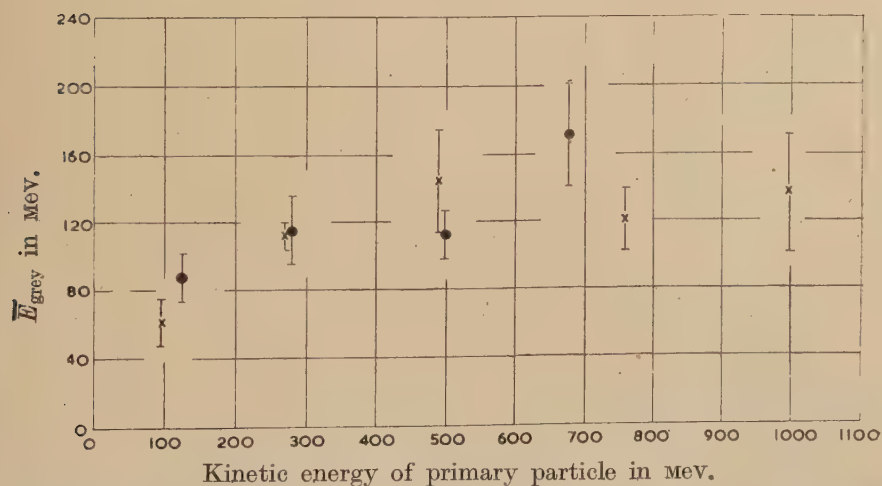
tions being made by the method described by Camerini *et al.* (1950, IV). The results are shown in tables 1 and 2, together with data obtained by other workers. The results for mesons are consistent with a geometrical cross-section for the nuclei of the emulsion. Some results for shower

Fig. 3



Variation with primary energy of the average number of 'grey' tracks, $\bar{N}_{\text{grey}}$, from meson (x) and proton (●) disintegrations.

Fig. 4



Average energy, $\bar{E}_{\text{grey}}$, of the particles producing the 'grey' tracks, as a function of primary energy for meson (x) and proton (●) disintegrations.

particles obtained subsequently to those given by Camerini *et al.* (1950, IV) are summarized in table 3. They are consistent with an interaction

Table 1. The Mean Free Path for Interaction of π -Mesons

Energy range (MeV)	Mesons	M.F.P. in g cm ⁻² of material	Observed M.F.P. Geom. value	Technique	Authors
30-50	π^-	95 ± 12, emulsion	1.5	Emulsion	{ Bernardini, Booth and Lederman (1951)
70-80	π^-	78 ± 10, emulsion	1.0		
60-90	π^-	86 ± 17, emulsion	1.1		
100-110	π^-	72 ± 7, emulsion	0.9		
30-40	π^-	86 ± 9, emulsion	0.93	do.	{ Bradner and Rankin (1950)
35-60	$\pi^\pm$	~50, Al	1.0	{ Cloud chamber	Camac <i>et al.</i> (1951 a)
55-70	$\pi^\pm$	~62, Al	1.2		
55-70	$\pi^\pm$	~100, C	1.7	do.	{ do. (1951 b)
85	π^-	125 ± 10, H	0.22	{ Counters	{ Chedester, Isaacs, Sachs and Steinberger (1951)
85	π^-	48 ± 2, Li	1.08		
85	π^-	59 ± 4, Be	0.96		
85	π^-	58 ± 2, C	1.08		
85	π^-	57 ± 2, O	1.2		
85	π^-	72 ± 3, Al	1.13		
85	π^-	106 ± 5, Cu	1.0		
85	π^-	117 ± 5, Cd	1.12		
85	π^-	143 ± 6, Pb	1.2		
80-1100	$\pi^\pm$	95 ± 28, emulsion	1.05	Emulsion	{ Lock and Yekutieli (1952)

Table 2. The Mean Free Path of Protons for Star Production

Stars	Energy range (mev)	$\bar{E}$	Path length (cm)	Estimated number of interactions	M.F.P. cm of emulsion	Authors
<i>a</i> $\{N_h \geq 1$	40-100	60	310	8.75	35 ± 13	Lock and Yekutieli (1951)
<i>b</i> $\{N_h \geq 3$	40-100	60	310	5.75	54 ± 20	
<i>c</i> Identified $\{N_h \geq 1$	100-500	180	304	10	30 ± 10	
<i>d</i> Protons $\{N_h \geq 3$	100-500	175	304	6	51 ± 20	
<i>e</i> $\{N_h \geq 1$	100-500	195	748	21	36 ± 8	Perry (1951)
<i>f</i> $\{N_h \geq 3$	100-500	195	748	16	47 ± 10	
<i>g</i> $N_h \geq 1$	—	240	—	281	82 ± 12	Germain (1951)
<i>h</i> $N_h \geq 3$	—	175	14470	91	160 ± 17	
<i>i</i> $N_h \geq 3$	—	245	9060	90	100 ± 11	
<i>j</i> $N_h \geq 3$	—	340	17500	206	85 ± 6	
<i>k</i> $N_b \geq 1$	350-400	375	—	404	56 ± 9	Bernardini, Booth, and Lindenbaum (1951)

The second group of Bristol results, lines *c* and *d*, refer to double stars caused by identical protons of track length greater than $3\,000\,\mu$, while the first and third groups refer to all double stars with an interconnecting track greater than $400\,\mu$ in length and of grain density within specified limits. The number of interactions due to protons has been estimated from the observed fraction of protons amongst the identified interconnecting particles of that grain density inter-

cross-section for shower particles between 0.6 and 1.0 times the geometrical value. The π -mesons, which comprise some 80% of the shower particles, must have an interaction cross-section of the same order of magnitude.

§3. CHARACTERISTICS OF THE PROTON DISINTEGRATIONS

Several studies have been made of the characteristics of 'stars' produced by protons of energy up to 385 mev (Bernardini, Booth and Lindenbaum 1951, Germain 1951, Perry 1951). The general features of the disintegrations appear to be in satisfactory agreement with the theories of Goldberger (1948) and Wouthuysen (1949). These theories assume that with protons of energy below 1 bev, the majority of the collisions are elastic in the sense that no energy is released in the form of mesons. Because of this feature it has been convenient to compare and contrast the meson induced stars with those due to protons.

Table 3. Some Recent Determinations of the Interaction Mean Free Path of Shower Particles

Method	Authors	M.F.P. in g cm^{-2} of material	Observed M.F.P.	Energy range (mesons)
			Geom. value	
Cloud chamber at mountain alti- tudes	Hartzler	230 ± 60 , Au	1.5	> 1000 mev
	Gregory and Tinlot	172 ± 30 , Pb 164 ± 50 , Al	1.1 2.0	$n_s \geq 3$
	Lovati, Mura, Succi and Tagliaferri	200 ± 50 , Pb	1.25	> 150 mev $n_s \geq 2$
Photographic emulsion technique	Lock and Yekutieli	120 ± 20 , emulsion	1.4	> 100 mev

(a) Meson Production by Fast Protons

In 60 of the 70 disintegrations due to protons the energy of the particle was greater than the threshold for meson production, viz. 170 mev. The ejection of a charged π -meson was observed in only two cases. Applying a correction for those mesons which escape identification,* it is found that only 0.067 ± 0.045 charged mesons are emitted per disintegration.

In an attempt to estimate the total number of mesons produced, both charged and uncharged, a study was made of the 'energy balance' in these disintegrations, for different values of the primary energy E_p . The average values of E_p , N_p , N_α , and N_{grey} were computed for each group,

* For details see § 5 (i).

together with the average energy for each of these types of particle. N_p represents the number of ejected protons, and N_α the number of ejected α -particles of energy less than 25 mev, i.e. those ejected during the evaporation process. In each group, except N_α , allowance was made for the emitted neutrons. The results for different values of E_p are shown in table 4. Although subject to large statistical errors, they strongly suggest that in the energy-range under consideration the fraction of the available energy which appears in the form of mesons is very small. It is, of course, possible that mesons are created and subsequently absorbed within the parent nucleus.

(b) *Recoil Protons*

The tracks of 52 secondary particles with a grain-density less than 80 grains per 50μ were found associated with the 70 stars due to protons. This value of the grain-density corresponds to a proton of 25 mev, a deuteron of 50 mev and a triton of 75 mev, respectively. Scattering and grain-density measurements showed that 2 of these particles were mesons, 17 were protons, 4 were deuterons and 1 was a triton. The others could not be identified.

Table 4. The 'Energy Balance' for Proton Disintegrations

Average energy of primary particle (mev)	Average energy in star (mev)
123 $\pm$ 15	133 $\pm$ 38
286 $\pm$ 11	204 $\pm$ 41
500 $\pm$ 12	410 $\pm$ 50*
681 $\pm$ 16	672 $\pm$ 188

* Allowance has been made for the two π -mesons observed to be ejected.

Let us assume that the primary proton, p , makes a succession of elastic collisions with individual nucleons in the nucleus. A cascade of recoiling nucleons will then be produced in the way illustrated schematically in fig. 5. For each ejected particle there is a trajectory through the nucleus which is linked with the primary collision. For example, particle 4 is ejected as a result of collisions a , b and c , i.e. after 3 collisions.

Suppose that the energy of the primary proton, E_p , the energy of each ejected charged nucleon, E_n , and the angle, α , between the direction of motion of the primary particle and that of the ejected nucleon are all known. If a nucleon is ejected after only one collision (e.g. particle 1), the relationship between E_p , E_n and α is

$$\cos \alpha = \sqrt{\{(1 + 2\mu/E_p)/(1 + 2\mu/E_n)\}}, \quad \dots \quad (1)$$

where μ is the proton rest-mass. If, in any particular case, the observed value of α is greater than that derived from the above expression, using the

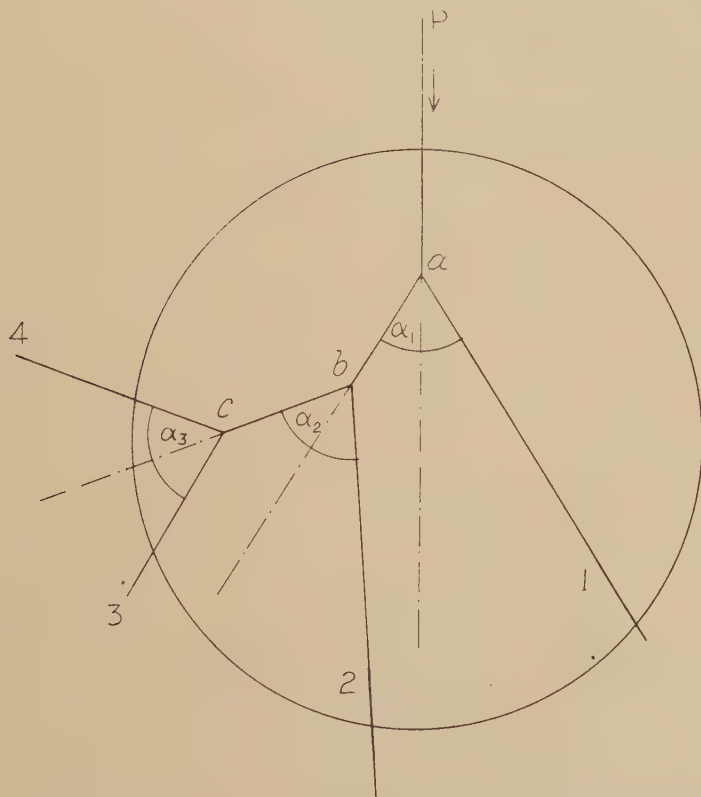
measured values of E_p and E_n , the particle must have been ejected as a result of more than one nucleon-nucleon collision. If the number of collisions leading to the ejection of a particle is m , the maximum value of α is obtained when the magnitude, β , of each of the m individual collisions is the same in each case, and equal to

$$\cos^{2m} \beta = (1 + 2\mu/E_p)/(1 + 2\mu/E_n) \quad . \quad . \quad . \quad (2)$$

and

$$\alpha_{\max} = m\beta. \quad . \quad . \quad . \quad (3)$$

Fig. 5



Schematic diagram of a nucleon initiated cascade in a nucleus.

Inserting the measured values of E_p , E_n and $\alpha(=\alpha_{\max})$ in expressions (2) and (3) we can find the minimum value of m , i.e. the minimum number of collisions required to cause the ejection of the observed particle with the measured values of E_n and α .

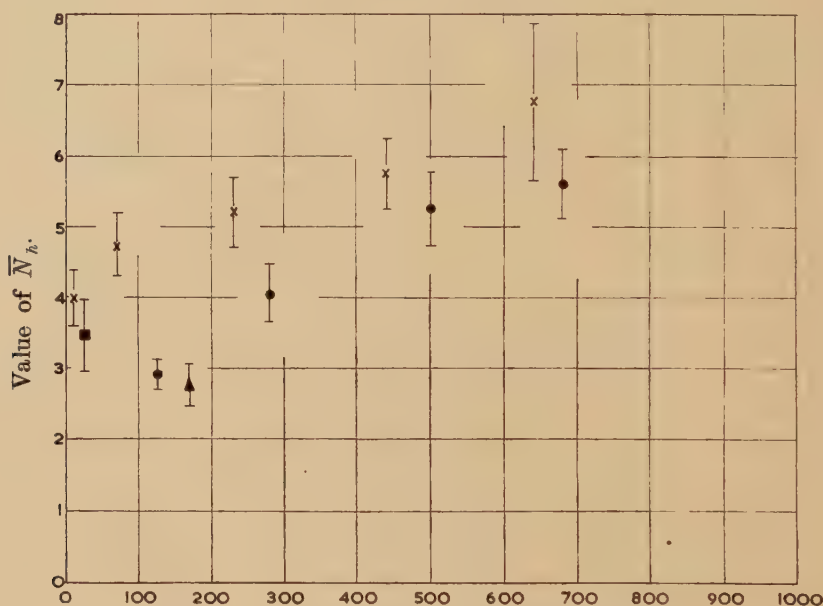
An analysis of the disintegrations from which 17 identified protons emerged showed that in 3 cases the proton could be regarded as having made only one collision in the nucleus (i.e. relation (1) was satisfied), and that in the remaining 14 cases at least two collisions took place. Similar

calculations have been made for the 28 unidentified secondary particles, assuming them all to be protons, and the results indicate a minimum of one, two and three collisions within the nucleus in 4, 17 and 7 cases, respectively. The average number of internal collisions—as deduced from all the 45 secondary particles—was 2.0, a value which corresponds to a cross-section for the interaction between two nucleons of $3 \times 10^{-26} \text{ cm}^2$.

(c) *Angular Distribution of 'Grey' Particles*

The angular distribution of the 'grey' tracks associated with the stars produced by protons is shown in fig. 7. The median angle of the distribution is $\sim 40^\circ$. For 240 mev protons, Perry (1951) finds that the median

Fig. 6



Variation of $\bar{N}_h$ with primary energy for proton (●) and 'meson-recoil nucleon' (×) stars.

■ Data from meson double stars.
▲ Data from proton double stars.

angle of the 'grey' particles is 30° . These figures are in qualitative agreement with the theoretical predictions of Goldberger (1948) and Wouthuysen (1949). (See also Bernardini, Booth and Lindenbaum 1951.)

§4. CHARACTERISTICS OF MESON DISINTEGRATIONS

The simplest model for the production of a nuclear disintegration by a π -meson has been proposed by Heitler and Janossy (1950). The incident meson 'knocks-on' a nucleon in the struck nucleus, and this recoil nucleon causes the observed disintegration by secondary collisions within the nucleus. We have attempted to test the validity of this model,

- (i) by comparing the variation of N_h with the energy of the primary particle for the meson and proton stars ;
- (ii) by studying the ejected protons ; and
- (iii) by studying the ejected mesons.

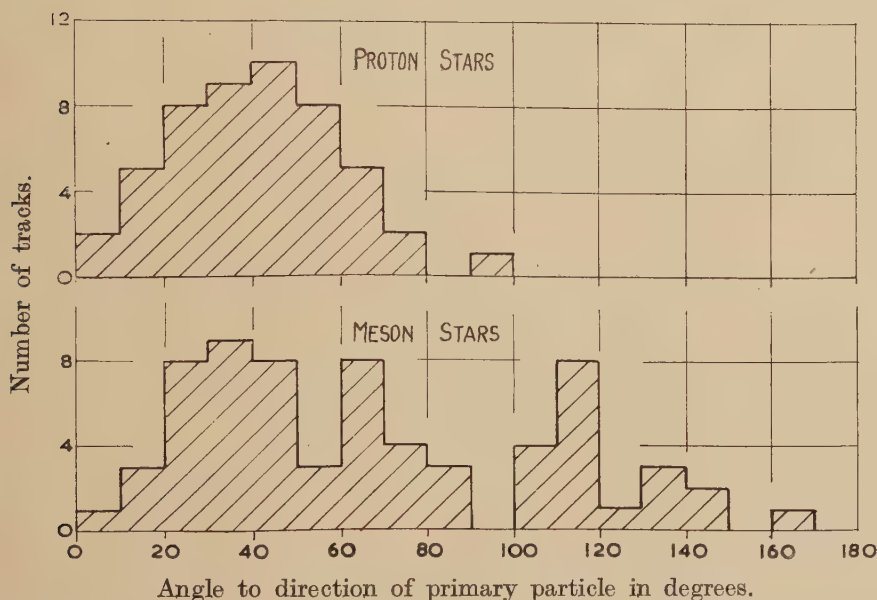
(i) *Variation of N_h with E_p .*

If the meson stars are indeed produced by a single recoiling nucleon, there should be a strong similarity between the stars produced by protons and by mesons. The maximum energy which a meson of total energy, ω_π , can transfer to a nucleon at rest is

$$E_{\max} = 2M_0(\omega_\pi^2 - m_0^2 c^4) / \{m_0^2 c^2 + M_0(2\omega_\pi + M_0 c^2)\} \quad . \quad . \quad . \quad (4)$$

(Janossy, 1950),

Fig. 7



Angular distribution of the 'grey' tracks from meson and proton disintegrations.

where m_0 and M_0 represent the rest-masses of the π -meson and proton, respectively. The nuclear disintegration produced by a recoil-nucleon of this energy will, on the average, be a little less energetic than one produced by a proton of the same energy incident from outside the nucleus, for the recoiling nucleon has to traverse a smaller thickness of nuclear matter.

For each disintegration produced by an identified meson, the maximum energy $E_{\max}$ which could be transferred to a recoil nucleon was computed from equation (4), and in fig. 6 the size distribution (N_h) of these stars is plotted as a function of $E_{\max}$. The distribution for disintegrations produced by protons is included in the same figure. There appears to be a significant difference between the two distributions, more energy being transferred to

the nucleus in the meson stars than is compatible with the assumption of a single recoiling nucleon. This result is particularly striking since we have assumed the recoil nucleon to receive the maximum and not the average energy. For low energies ($\omega_\pi < 450$ mev), the energy transfer is large compared with that of a proton of the same energy (see fig. 2) and it is reasonable to assume that a considerable fraction of the mesons are completely absorbed in the nucleus. (See also § 5, iii.)

(ii) *Recoil Protons*

69 particles of which the grain density in the track was less than 80 grains per 50μ were emitted from 66 stars produced by π -mesons of energy greater than 160 mev. 23 of these particles were protons, 2 were deuterons, and the others could not be identified.

If we assume that the incident meson makes only one collision with a nucleon when it interacts with a nucleus, then all the ejected nucleons will arise from the nucleon 'cascade' produced by the recoil particle. This cascade will be similar to that initiated by each of the recoiling particles arising from the first nucleon-nucleon collision in the disintegrations produced by protons.

Now the number of collisions which link the ejection of a charged nucleon with the first meson-nucleon collision may be calculated in a way similar to that described in § 3(b). It is thus found that the average number is about two. On the other hand, in disintegrations due to protons, the first generation of recoil particles were found to make, on the average, only one collision before escaping from the nucleus. It is reasonable to suppose that the physical processes responsible for the nucleon cascades, in the disintegrations produced by protons and mesons, are similar. The above result therefore appears to be inconsistent with the assumption of a single meson-nucleon collision in the disintegrations produced by mesons. The results suggest that only about 30% of the emerging nucleons are ejected as a consequence of the first meson-nucleon interaction.

Let λ be the average number of collisions which the incident meson makes in traversing the nucleus. The probability that the meson makes one or more collisions is $(1 - e^{-\lambda})$, and the fraction of the total number of meson collisions which correspond to the first meson-nucleon interaction is $(1 - e^{-\lambda})/\lambda$. Since the results given above show that about 30% of the meson-nucleon collisions correspond to these encounters, it follows that the average number of collisions which the incident meson makes in traversing the nucleus is about three.

(iii) *Angular Distribution of the 'Grey' Particles*

Fig. 7 shows the angular distribution of the 'grey' tracks associated with the meson disintegrations. Any tracks identified as due to mesons have been excluded. The distribution is much broader than that for proton disintegrations, and a considerable number of the particles are ejected at an angle of more than 90° to the primary meson. The fraction of

such particles is called the 'backward fraction', B_g , and its observed value is 0.29 ± 0.05 . The median angle of the distribution is 60° . It is possible that some of the particles ejected backwards are unidentified π -mesons, but their number cannot be very great, for the value of B_g for the identified protons amongst the 'grey' particles is 0.21 ± 0.10 . The ejection of these particles at a large angle to the direction of the primary meson cannot be explained in terms of a single meson-nucleon collision.

(iv) Summary

The results show that the meson disintegrations cannot be interpreted in terms of a single nucleon, recoiling from the incident meson, and producing a nucleon cascade. The average number of meson-nucleon collisions within the nucleus, for nuclei of average atomic number, appears to be between two and three. In collisions with nuclei, mesons of low kinetic energy, < 160 mev, have a high probability of being absorbed.

§ 5. THE FATE OF THE INCIDENT MESON

As a result of its interaction with a nucleus an incident meson may

- (i) emerge as a charged meson, having lost some energy;
- (ii) emerge as a neutral meson, having suffered charge exchange;
- or (iii) be completely absorbed, the energy corresponding to its rest-mass contributing to the disintegration of the nucleus.

(i) Ejection of a Charged Meson

In 16 cases a single charged π -meson has been observed to emerge from a meson-disintegration, and the results are summarized in table 5. In most of these events the energy of the outgoing meson is much less than that of the primary particle. There appears to be no simple correlation between the loss of energy and the angle of scattering of the meson.* A similar observation has been reported by Bradner and Rankin (1951) for 38 mev π^- -mesons, and by Bernardini *et al.* (1951a, b) for 85 and 110 mev π^- -mesons.

Energy and momentum considerations show that the first three cases given in table 5 may be attributed to a single scattering of the primary meson by a nucleon. Similarly, the fourth may be attributed to the scattering of a meson by an α -particle. In each instance, the kinetic energy of the primary meson was less than 350 mev.

Assuming that a π -meson can only lose energy in elastic meson-nucleon collisions, the observed energy loss in the remaining 12 cases can only be accounted for if it is assumed that the average number of collisions, made by the meson with nucleons, is about three. The minimum number of collisions in each individual case is given in table 5.

The 16 cases given in table 5 represent favourable examples, in which, by chance, the track length in the emulsion was sufficiently great to permit identification. An estimate of the fraction of ejected mesons which are not identified has been made in the following way: of tracks with

* For plates see end of issue.

grain-density less than 80 grains per 50μ a certain number have been identified as mesons, protons, deuterons or tritons. It is reasonable to assume that the proportions of different particles amongst those not identified are the same as for the identified particles. An approximate

Table 5. The Scattering of Energetic π -Mesons

Number	Kinetic energy of primary meson† (mev)	Kinetic energy of secondary meson† (mev)	Angle of scattering	'Grey' particles* (mev) (angle)	N_h	Minimum number of collisions
1-07	79	67	138°	— —	2	1
1-19	50	39	63°	— —	1	1
2-07	252	180	62°	— —	4	1
2-16	340	28	28°	— —	1	1
3-03	460	310	26°	— —	1	1 (2)
3-14	640	39	18°	380 7°	7	3
3-15	375	85	19°	— —	6	2
4-07	755	70	167°	— —	6	2
4-11	755	450	15°	42 18°	6	1 (2)
4-17	825	17	125°	100 82°	5	4
4-19	670	110	22°	— —	9	2
4-21	785	56	96°	— —	6	3
5-01	900	29	43°	45 104°	6	3
5-08	1040	120	5°	52 20°	3	2
5-11	960	225	17°	— —	3	2
5-12	1050	120	90°	— —	4	2

* In this column the energy of the 'grey' particle is given first, assuming it to be a proton, followed by the angle which it makes with the primary particle.

† The error in the values of the energy is $\leq 20\%$.

Table 6. The Frequency of Ejection of π -Mesons from Meson Disintegrations

Group	Primary energy (mev)	No. of stars	No. of mesons observed	$\bar{n}_\pi$, correcting for loss
I	0-160	20	2	0.265 ± 0.115
II	160-360	18	2	0.21 ± 0.15
III	360-660	15	3	0.43 ± 0.17
IV	660-860	21	5	0.55 ± 0.16
V	860-1060	11	4	0.61 ± 0.25
II-V	160-1060	65	14	0.44 ± 0.12
All	0-1060	85	16	0.39 ± 0.10

value can then be obtained for the total number of charged mesons, $\bar{n}_\pi$, ejected from meson disintegrations, for different values of the primary energy. The results are shown in table 6. The value of $\bar{n}_\pi$ for all the stars is 0.39 ± 0.10 .

(ii) *Conversion*

It follows from the above result that $\sim 60\%$ of the incident mesons are either absorbed in the target nucleus, or emerge from it having lost their charge. Evidence for the relative importance of these two processes can be obtained by studying the energy-balance in the meson disintegrations. If the average energy associated with a sample of such disintegrations is greater than the average kinetic energy of the primary particles, $\bar{E}_p$, this is evidence for the absorption of mesons; if it is less, it is reasonable to attribute the 'missing energy' to charge-exchange, for the neutral π -mesons commonly escape detection; and if there is a balance of energy it may be that both absorption and charge-exchange occur, the energy liberated by absorption compensating that 'lost' through conversion.

The study of energy-balance has been made by the same methods employed for the proton stars. Allowance has been made, however, for the ejection of charged mesons, using the results given in table 6. The results are set out in table 7, from which it may be seen that for mesons of

Table 7. The 'Energy Balance' for Meson Disintegrations

Average energy of primary particle (mev)	Average energy in star (mev)
95 ± 7.5	129 ± 22
270 ± 12	206 ± 37
493 ± 20	568 ± 190
761 ± 13	605 ± 91
1000 ± 15	771 ± 183

energy less than 160 mev there is some evidence for absorption. This result will be considered in more detail in the next section. For mesons of energy greater than 160 mev there is some indication of 'missing energy' and therefore, of charge exchange.

The conservation of energy may also be expressed, for these disintegrations, by the relation:

$$140 + E_p = E_{\text{star}} + 140\bar{n}_\pi + \bar{n}_0(140 + \bar{E}_\pi), \quad (5)$$

where E_p represents the kinetic energy of the primary meson in mev; E_{star} , the total kinetic energy in the star including that of the neutrons and charged mesons; $\bar{E}_\pi$ the average kinetic energy of the ejected charged π -mesons, and also, it is reasonable to assume, of the ejected neutral mesons; and $\bar{n}_0$ = average number of ejected neutral mesons.

Solving for $\bar{n}_0$, we obtain

$$\bar{n}_0 = \left\{ \left(\frac{E_p - E_{\text{star}}}{140} \right) + (1 - \bar{n}_\pi) \right\} / \left(1 + \frac{\bar{E}_\pi}{140} \right). \quad (6)$$

We have calculated $\bar{n}_0$ for three ranges of primary energy, viz. 0–160 mev, where it is already known that some absorption occurs, 160–800 mev and 800–1100 mev. The results are given in table 8. Although subject to large statistical errors the figures give an indication of the relative probabilities for the different processes which can take place in meson–nucleon collisions. It can be seen that approximately 50% of the incident mesons suffer charge exchange, irrespective of their energy.

It is possible, in principle, to observe the electron pairs arising from the gamma rays which are the decay products of the neutral π -mesons. However, the frequency with which such conversion electrons appear near the parent star must be small, and a search for them would be very arduous in the present experimental conditions.

Table 8. The Relative Probabilities for the Scattering (Conversion or Absorption of Energetic π -Mesons

Energy range (mev)	Scattering $\bar{n}_\pi$	Conversion $\bar{n}_0$	$\bar{n}_\pi + \bar{n}_0$	Absorption $1 - (\bar{n}_\pi + \bar{n}_0)$	Multiple processes
0–160	0.26	0.38	0.64	0.37	—
160–800	0.40	0.57	0.97	—	—
800–1100	0.75	0.70	1.45	—	$\sim 0.45?$

The statistical errors associated with the values of $\bar{n}_\pi$ have been given in table 6; those associated with the values of $\bar{n}_0$ are of the same order of magnitude, i.e. $\sim 30\%$.

(iii) Absorption

It is difficult to prove the absorption of mesons of great kinetic energy because the contribution from the rest-mass is relatively small. The results shown in table 8 indicate, however, that no absorption takes place at energies above 160 mev. For mesons of kinetic energy less than 160 mev, the absorption of a meson can be definitely established in three disintegrations as a result of the ejection of a proton with a kinetic energy greater than that of the primary meson. Assuming that an equal number of fast neutrons are ejected in a similar manner, the fraction of mesons absorbed in this energy range is $\sim 50\%$. This is in agreement with the value derived in table 8, and with the data of Bernardini *et al.* (1951), who have shown that when a beam of 110 mev π^- -mesons passes through matter about 65% of the particles are completely absorbed by the nuclei which they encounter.

(iv) The Multiplication of Mesons

We have observed one event in which two mesons, of kinetic energy 365 and 375 mev respectively, were ejected from a nucleus disintegrated by a π -meson, of 1050 mev kinetic energy. Energy considerations suggest that the production of the extra meson (one presumably being the primary

particle with reduced energy) cannot be explained in terms of meson production by a nucleon 'knocked on' by the primary meson*†. However, the event may be explained if it is assumed that π -mesons can create further mesons directly in collisions with nucleons. The cross section for this process of 'meson splitting', at energies of the order of 1 bev, has been estimated by Heitler and Janossy to be about 1/10 of that for the large angle scattering.

Further evidence for the reproduction of mesons in meson-nucleon collisions has been put forward by Rosser and Swift (1951). They have exposed photographic emulsions above and below 30 cm of lead, at an altitude of 2 860 m, and find that for showers of $n_s \geq 2$ the ratio of charged to neutral primary particles is much greater for the showers observed under the lead, than for those above. This result is confirmed by similar observations made from plates exposed at an altitude of 3 540 m (Camerini, Lock and Yekutieli 1951). Rosser and Swift suggest that the increase in the number of showers with charged primaries is caused by π -mesons created in the lead assembly, which produce showers of π -mesons.

CONCLUSIONS

If the disintegrations produced by fast π -mesons are to be explained in terms of elastic meson-nucleon collisions, the average number of such collisions in the nucleus must be between two and three. This result is obtained from, (i) a comparison of the size distribution of meson and proton stars; (ii) an analysis of the recoil-protons in meson and proton stars; and, (iii) a study of the characteristics of the secondary mesons which emerge from the collision. These observations refer mainly to stars with $N_h \geq 3$. However, the geometrical cross-section for the meson-nucleus interaction, derived from an analysis of the double stars, is consistent with this average number of collisions. On the other hand, the meson-nucleon cross section of $1.3 \times 10^{-26} \text{ cm}^2$ obtained by Chedester *et al.* (1951) for 85 mev π^- -mesons corresponds to only one collision per nucleus.

In the low energy range ($E_p < 160 \text{ mev}$) there is direct evidence for the absorption of about half the mesons which collide with nuclei. Absorption does not appear to occur at higher energies. At all values of the primary energy about 50% of the incident mesons suffer charge exchange. A definite case of 'meson splitting' has been observed at an energy of the order of 1 bev, but the available data does not allow an estimate to be made of the frequency of occurrence of this type of multiple process.

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* For a further discussion of this event see Camerini *et al.* (1951, VII).

† Note added in proof.—We have recently observed a second event of a similar nature.

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XIX. *A Scintillation Spectrometer for High-Energy Gamma-Rays*

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SUMMARY

This paper describes a scintillation spectrometer, which measures the energy of the pairs produced by gamma-rays. Each gamma-energy gives a symmetric peak in the energy distribution. The width at half-maximum is 7% at 4 mev. The greatest advantage of this apparatus is the high sensitivity. It has been possible to measure sources with a strength of about $0.1 \mu\text{c}$.

§1. INTRODUCTION

IN measuring high-energy gamma-rays with a one-crystal scintillation spectrometer a complicated pulse height distribution is obtained when the crystal is a substance containing elements of high atomic number, as, for instance, thallium-activated sodium iodide. For every gamma-energy a photoelectron peak and a continuous Compton distribution are obtained. The electron-positron pairs of each gamma-energy cause several peaks in the distribution. When the annihilation radiation from the positron is captured by the crystal the entire energy of the gamma quantum is given off to the crystal. If one or both annihilation quanta escape the crystal, the energy is reduced by 0.51 and 1.02 mev, respectively. Pair production thus gives rise to three peaks for each gamma-energy in the pulse height distribution. One of these peaks coincides with the photoelectron peak. Gamma-rays with several energies therefore produce a very complicated pulse height distribution and in such cases accurate measurement of the energy can be very difficult. Naturally a substance with elements of only low atomic number may be used as the crystal, for example anthracene. The photoelectric effect and pair production have then a very low cross section and the pulse height distribution consists solely of Compton distributions. The difficulty then arises of determining the end point of a continuous distribution, which is especially hard in this case, on account of the comparatively low resolution of the scintillation spectrometer.

It is possible, however, to employ the two annihilation quanta emitted from the positron to separate the pair production from the other processes in a sodium iodide crystal. The counter which is used for measuring the energy is surrounded in two opposite directions by two other counters

* Communicated by Professor S. von Friesen.

designed to capture the annihilation radiation. Coincidences between these two counters are used to gate the pulses from the central counter. The pulse height analyser thus gives the distribution of those pulses in the central counter which are simultaneous with pulses in the two other counters, so that the distribution of the pulses due to the electron-positron pairs is obtained. The pairs from a certain gamma-ray all give off the same energy to the crystal, and each gamma-energy thus corresponds to a symmetric peak in the pulse height distribution. This facilitates a careful measurement of energy and also a study of gamma-rays with several energies.

A few measurements undertaken with such an apparatus were described in a previous report (Johansson 1950 b). The present paper describes more fully an improved apparatus constructed according to the same principle. The idea of using annihilation quanta to select the pair production pulses has also been advanced by Hofstadter and McIntyre (1950). Bair and Maienschein (1951) have quite recently published measurements with a similar apparatus.

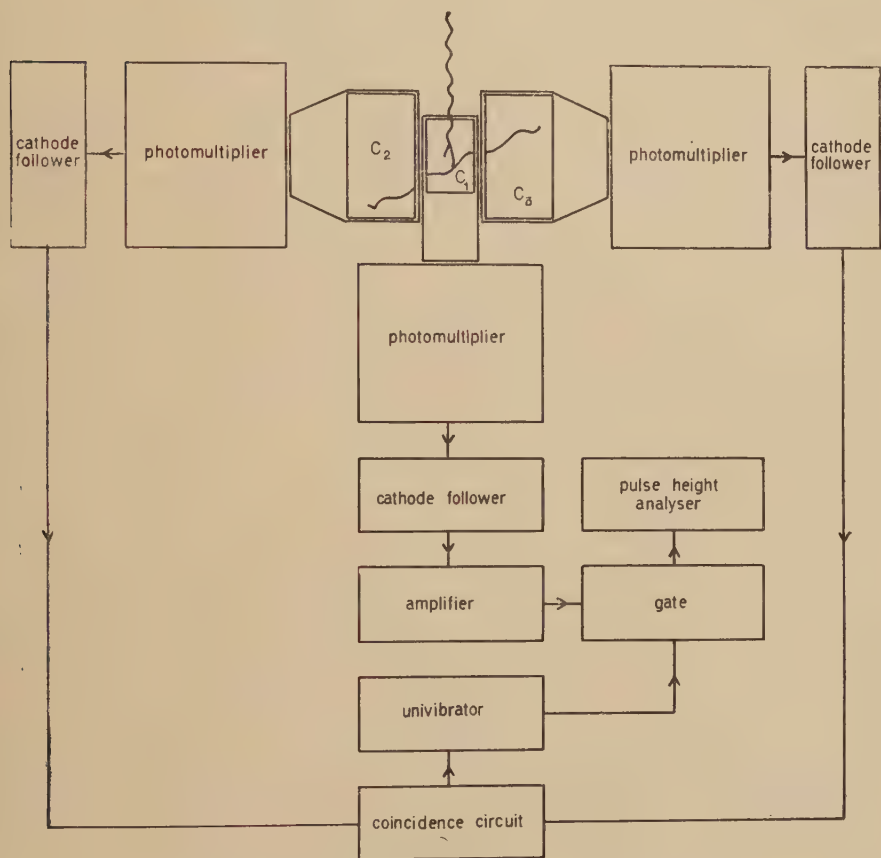
§ 2. APPARATUS

Fig. 1 shows a schematic diagram of the apparatus. A central crystal (C_1) with the dimensions $13 \times 13 \times 20$ mm is surrounded by two side crystals (C_2 and C_3) with the dimensions $35 \times 35 \times 20$ mm. Each of the three sodium iodide crystals is enclosed in a container. The containers are nickel-plated on the inside and closed at one end by a window of perspex, which lies close to the top of the photo-multipliers. The multipliers are of type E.M.I. 5311. The pulses from the multipliers are coupled to three cathode followers, which are fastened to the box and transmitted further by means of cables to other parts of the apparatus. The pulses from the central counter are amplified in a linear amplifier with negative feedback and are coupled to a gate. The side counters are connected to a coincidence circuit and the coincidence pulses trigger a univibrator which opens the gate. The gated pulses, finally, are analysed in a pulse height analyser.

The size of the crystals depends on several factors. The central crystal should be comparatively large in order to obtain a high degree of sensitivity. Another reason which speaks in favour of a large crystal is that the range of the electrons and positrons must be only a small part of the linear dimensions of the crystal. Otherwise it may occur that a fairly great number of pairs give off only part of their energy to the crystal, thus causing a deformation of the peak in the pulse height distribution. The central crystal must not, on the other hand, be made so large that a greater part of the annihilation radiation is captured in the crystal, as this would produce reduced sensitivity. The two side crystals should be made as large as possible so as to capture the largest possible part of the annihilation radiation. This is essential for the sensitivity of the apparatus.

Only those pulses from the central counter which are due to pair production are accompanied by simultaneous pulses in the two side counters. The pulses which reach the pulse height analyser, therefore, are a measure of the energy of an electron-positron pair. They give a peak in the pulse height distribution. The two side counters naturally do not capture only the annihilation radiation but also scattered rays. If the incident rays are not collimated these counters also count the radiation direct from the source. They may therefore attain a rather

Fig. 1



Schematic diagram of the scintillation spectrometer.

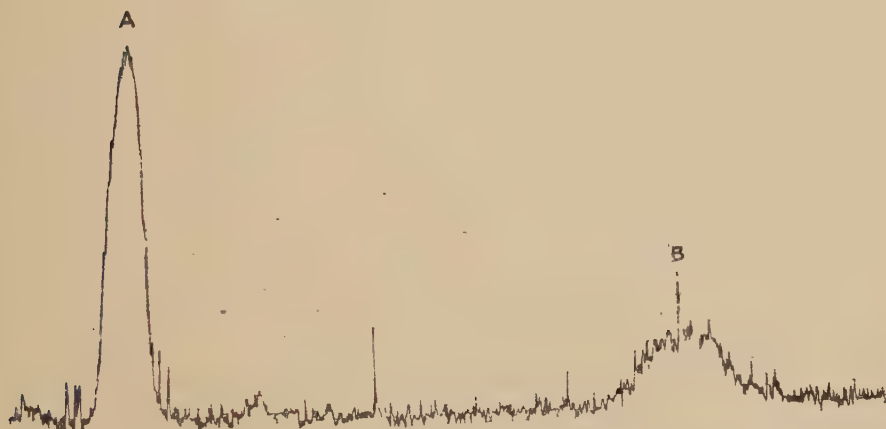
high counting rate. Accidental coincidences may then open the gate for all kind of pulses from the central counter. In this way one may obtain a continuous distribution in the pulse height distribution as a background to the pair peak. In order to avoid this, weak sources should be used or the rays be collimated. The coincidence circuit should also have a short resolving time. The circuit used in the present work is of a type described by Garwin (1950). In order to make the apparatus as

stable as possible the coincidence circuit was not set at the highest possible resolution, 10^{-7} sec proving quite sufficient. The pulses from the cathode followers are shaped with delay-lines, short-circuited at one end. The coincidence circuit is driven directly from the cathode followers without amplification. This is possible since the voltage of the two multipliers of the side counters is comparatively high, about 2 000 v. The voltage of the multiplier of the central counter, on the other hand, is low, only about 1 000 v. The relation between energy and pulse height has been shown not to be linear at higher voltages (Johansson 1951).

The gate and the pulse height analyser may be constructed in three different ways. The simplest one is, perhaps, to pass the pulses from the central counter to the vertical deflection plates of a pulse oscilloscope. The pulses from the coincidence circuit are then made to start the sweep. The oscilloscope then acts as a gate. On the screen only the pair production pulses are seen and they start from one and the same point. They have a very short rise time, and what is seen is the exponentially decreasing part. If the counting rate is sufficiently high the pulse height distribution can be studied directly on the screen. It is best, however, to have a photographic record of the screen. The time of exposure is chosen so that a large number of pulses is photographed. Since all the pulses start from the same point of the screen the images on the plate are superimposed together and on a suitable adjustment of the brightness a picture is obtained where the density is a measure of the number of pulses. The oscilloscope is of the type Tektronix 511 A. The sweep speed was $0.1 \text{ cm}/\mu\text{sec}$. As collimator lens in the photographing a Zeiss Sonnar $f:1.5$ was used. The counting rate was usually about 1 pulse/second and the time of exposure 10 minutes. Each gamma-ray energy gives a diffuse band on the plate, corresponding to a peak in the pulse height distribution. In addition, the zero line is reproduced, due to the fact that the accidental coincidences trigger the sweep without the simultaneous occurrence of a pulse on the vertical deflection plates. If an unknown gamma-spectrum and a line of reference are photographed on the same plate the pulse heights can be compared directly. Since all the pulses have the same time constant, this comparison can be made along an arbitrary line at right angles to the zero line. It is better, however, to determine the density along a line at right angles to the zero line by means of a microphotometer. Fig. 2 shows such a densitometer trace for gamma-rays from a RdTh -source with a strength of 0.1 mc and placed at a distance of 20 cm from the crystals. The rays were collimated through 16 cm of lead. In the figure the zero line (A) together with a well-defined peak (B) are seen. This peak is due to the gamma-ray of 2.62 mev from ThC'' . From a densitometer trace of this kind the pulse height can be determined with good accuracy. The pulse height is proportional to the energy and therefore the energies of gamma-rays can be measured after calibration with some known gamma-energy. Measurements of a few well-known gamma-energies agreed well with the accepted values.

On many occasions it may be necessary to determine the height of single pulses from the spectrometer, for instance, when extremely weak sources are used and when a quantitative comparison of the intensity of various energies is desired. The spectrometer is therefore equipped with an electronic pulse height analyser. The pulses from the coincidence circuit trigger the univibrator, which gives rectangular pulses with a length of $1\text{ }\mu\text{sec}$. These pulses open an electronic gate for the pair production pulses. The rise time of the latter is very short and the time constant comparatively long ($100\text{ }\mu\text{sec}$). When the gate is closed after $1\text{ }\mu\text{sec}$ they are cut off, becoming rectangular in shape with an almost flat peak. This form enables the use of a fairly simple single-channel differential discriminator as the pulse height analyser. The latter consists

Fig. 2



Densitometer trace showing the pulse height distribution of a RdTh-source.

of two Schmitt triggers together with an anti-coincidence circuit. The bias voltages are set by means of a potentiometer. A constant difference between the triggering levels is obtained by means of batteries. This device thus counts the number of pulses in a certain pulse height interval of variable size and position. The pulses from this analyser, finally, are counted with a conventional scaler and a register.

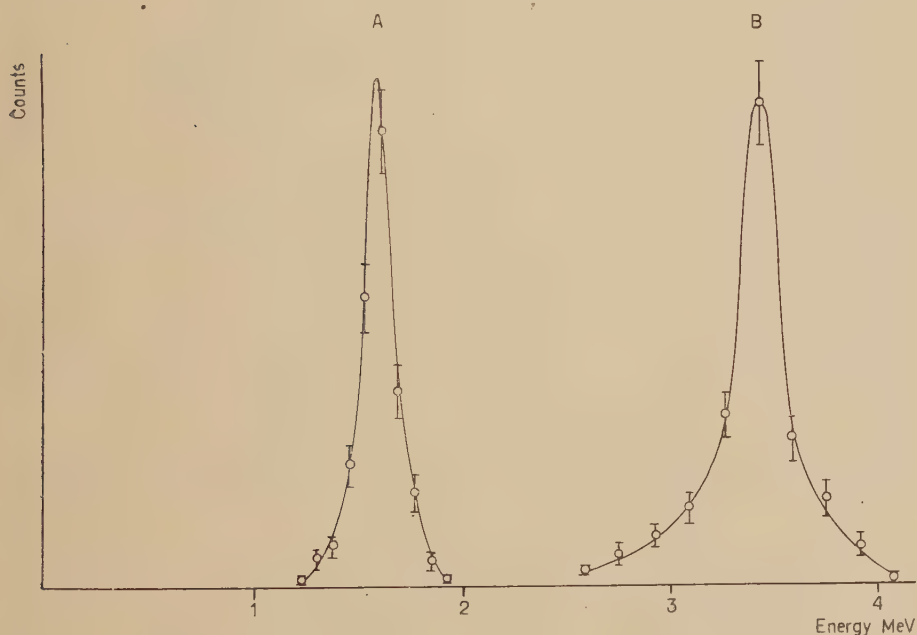
The oscilloscope may be used as a second means of measuring the height of single pulses. The pulses from the central counter, which in this method have a time constant of $1\text{ }\mu\text{sec}$, are stretched by means of a pulse stretcher to $5\text{ }\mu\text{sec}$. The stretcher consists of a delay-line, short-circuited at one end and charged via a number of crystal rectifiers (Craib 1951). The pulses obtained are almost rectangular in form with a flat peak. They are coupled to the vertical deflection plates of the oscilloscope and the latter is set for recurrent sweep. The sweep speed is kept fairly low

(0.1 mm/ μ sec). The peaks of the pulses are then seen as bright spots on the oscilloscope screen. The brightness is then lowered by means of the potentiometer regulating the grid voltage of the cathode-ray tube so that pulses are no longer seen on the screen. The rectangular pulses from the univibrator are set at a length of 5 μ sec and are coupled to the cathode of the cathode-ray tube. They have a negative sign and the brightness of the screen has therefore an increased value for a time of 5 μ sec. Thus on the screen only those pulses from the central counter are seen which are accompanied by a simultaneous pulse from the coincidence circuit. The brightness of the cathode-ray tube acts as a gate and only the pair production pulses are visible on the screen. As in the preceding case, a number of accidental coincidences are also obtained with the result that a number of bright spots appear along the zero line when the intensity is increased without the simultaneous appearance of a pulse on the vertical deflection plates. If the screen is photographed, the peak of each pulse appears as a spot on the plate. The height of the pulses is obtained by measuring the distance between these spots and the zero line. The time of exposure is made so long that the greatest possible number of spots are obtained, not so many, however, that they coincide. In order to get a sufficient number of pulses for the measurements it may be necessary to make several recordings. As a rule a satisfactory picture of the pulse height distribution can be obtained by measuring 3 to 4 plates.

The last two methods give results that agree well. Fig. 3 shows a pulse height distribution that is representative for both methods. It shows the number of pulses as a function of the energy. The central counter was calibrated prior to the measurements with some well-known gamma-rays as previously described (Johansson 1950 a, 1951). The energy seen in the figure is the energy of the pairs (the energy of the gamma-rays minus 1.02 mev). The peak A is due to a RdTh-source. The strength of the source was 0.1 mc, and it was placed at a distance of 20 cm from the crystals. The radiation was collimated through 16 cm of lead so as to avoid too high a counting rate in the side counters. When weaker sources are used measurements can be made without collimation although they require more time. The main part of the gamma-rays from the source has an energy of 2.62 mev. Other lower energies do not cause any peaks in the distribution, partly on account of lower intensity and partly on account of the great decrease in the cross section of the pair production at lower energies. The peak B comes from a Po-Be-source. In the nuclear reaction of ${}^9\text{Be}(\alpha, n){}^{12}\text{C}$ high energy gamma-rays are emitted. According to the latest measurements their energy is 4.45 mev (Pringle *et al.* 1950, Bradford and Bennet 1950, Bell and Jordan 1950). The source emitted a total of approximately 10^4 quanta per second. It was mounted close to the central crystal without collimation. The energy was determined to be 4.45 ± 0.05 mev, which agrees well with previous measurements.

The spread of the pulse height around a mean value is due largely to statistical fluctuations in the number of electrons emitted from the photo-cathode of the multiplier. The statistical fluctuations in the gain of the multiplier also contribute to the spread. Theoretically, the peaks should have the form of Gaussian distributions with the widths at half-maximum proportional to the root of the energy. The peaks in fig. 3 are fairly easily approximated as Gaussian distributions. The widths are 10 and 7%, respectively. The relation between these figures is on the whole what might be expected theoretically.

Fig. 3



Pulse distribution of a RdTh-source (A) and a Po-Be-source (B).

§ 3. DISCUSSION

The apparatus described in the foregoing measures the energy of the electron-positron pairs produced by gamma-rays and is thus a type of pair spectrometer. It may be of interest to compare it with a magnetic pair spectrometer, for example the one described by Walker and McDaniel (1948).

According to what has been said above the resolution expressed as the width at half-maximum of the apparatus described here is 7% at 4.4 mev, improving with increased energy. At 10 mev it should theoretically be 4%. The value for the magnetic pair spectrometer is given as 6%. The two types of spectrometer have thus approximately the same resolution.

The lower limit of the scintillation spectrometer is determined by the cross section of the pair production. At too low energies the measurements take a long time. The decrease in the resolution, on the other hand, is less significant. The peaks in the pulse height distribution retain their symmetrical shape, allowing a fairly accurate energy determination also at low energies. The lower limit for the spectrometer probably lies around 1.5 mev.

The greatest advantage of this type of scintillation spectrometer is its great sensitivity. No collimation is necessary and the crystal captures a large part of the incident rays. Even very weak sources can therefore be measured. The Po-Be-source used in the present measurements emitted a total of approximately 10^4 high-energy gamma-rays per second. The counting rate of the spectrometer was then 1 pulse per second, i.e. 10^{-4} of the gamma-quanta emitted were counted. At higher energies the sensitivity is increased, due to the increasing cross section of the pair production. For the magnetic spectrometer the peak counting rate per single counter pair is reported to be 10^{-7} of the total emitted quanta at 17 mev. The scintillation spectrometer is thus considerably superior with regard to sensitivity. For practical reasons it may sometimes be impossible to place the source as close to the crystal as in this measurement with a Po-Be-source. If the source is moved 5 cm from the crystal the sensitivity decreases with a factor of 10.

The gamma-spectrometer described in this paper is fairly simple to construct. Its great sensitivity makes it useful in studies of high-energy gamma-rays of low intensity, for example from nuclear reactions.

ACKNOWLEDGMENTS

The author wishes to thank Professor B. Edlén and Professor S. v. Friesen for their kind interest in this work.

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XX. CORRESPONDENCE

Some Observations on Slip Lines in Iron

By H. W. PAXTON, M. A. ADAMS and T. B. MASSALSKI
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[Received November 26, 1951]

GREENLAND (1937) was the first to observe that the appearance of slip lines on mercury single crystals depended on previous mechanical straining. Brown and Honeycombe (1951) have recently studied the slip lines in single crystals of the face centred cubic materials aluminium, copper, lead and gold, and shown that the form of these lines is sensitive to the state of preparation of the surface.

The purpose of this note is to point out that slip lines on the polished surface of body centred cubic iron single crystals behave similarly. The crystals were grown from decarburized mild steel (Si 0.1%, S 0.05%, P 0.05%, Mn 0.3%) in the form of wire 2 mm in diameter using the strain anneal method. About 0.003% C was introduced afterwards, and so the crystals showed the yield phenomenon (Cottrell and Churchman 1949, Schwartzbart and Low 1949). The gauge lengths were approximately 5 cm and the crystals were extended with a strain rate of 2×10^{-4} per minute in a rigid beam loading apparatus. For testing, each crystal was soft soldered axially into mild steel endpieces attached to the loading shackles through flexible Bowden cable and ball seatings. The upper and lower lengths of Bowden cable were of opposite twist to eliminate rotational torque.

Fig. 1 (Plate VII†) is a typical illustration of the type of slip lines produced by an extension of 0.5% on a mechanically polished surface or on an electropolished surface which had been lightly rubbed with fine polishing alumina. This extension was sufficient to take the specimen through the yield point. The separation of lines is of the order 5–10 μ . If, however, the crystal face was given a preliminary electropolish (using a mixture of perchloric acid and acetic acid) after grinding the metallographic surface, the slip lines that can be produced by 0.5% extension are extremely difficult to see with an optical microscope and much closer together, as shown in fig. 2 (Plate VII). The separation is approximately 2 μ .

It is necessary to polish electrolytically to beyond a certain depth (about 10 μ in the present specimens), presumably to remove the cold worked layer, if the coarse slip lines are to be avoided. When the electropolishing is not as deep as this the specimen appears as in fig. 3 (Plate VII).

* Communicated by Professor A. H. Cottrell.

† For plates see end of issue.

Here both types of slip occur on the same surface, the coarse slip being limited to following a straight band across the crystal which is the trace of a polishing scratch. It should be noted that the two types of slip are continuous, the coarse lines branching at the boundary of the band to form the fine ones.

Fig. 4 (Plate VIII) is a further photograph of this effect, taken slightly out of focus to show the traces of two scratches intersecting. The coarse slip lines appear on the grooves: the fine slip lines are not in focus.

In fig. 5 (Plate VIII), the approach of coarse slip lines to a grain boundary is shown. Here the coarse slip branches about 20μ from the boundary and no visible slip is seen at all within about 5μ from it. Slip took place in the other crystal, but was not in the right orientation to be shown in this photograph.

The authors wish to express their appreciation to Professor A. H. Cottrell for helpful discussion. They are also grateful to Professor D. Hanson and the University authorities for provision of facilities and financial support.

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An Attempt to Produce a Thermo-Nuclear Reaction in Deuterium by Means of a High Current Spark Discharge

By P. REYNOLDS and J. D. CRAGGS
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[Received November 21, 1951]

THE two reactions in deuterium given below have a high probability of occurrence, and are therefore suitable for an attempt to produce a thermo-nuclear reaction.



According to Gamow and Critchfield (1949), in order that these reactions should proceed at an observable rate, a temperature in the region of 10^6 °C, and a degree of ionization of the gas of nearly 100% are required.

A generator had been built in this laboratory for a general investigation of the physical conditions in high current transient discharges, and it was thought to be worthwhile to investigate the effect of passing sparks

* Communicated by Professor H. W. B. Skinner, F.R.S.

carrying currents in the region of 300 kA through deuterium. It was hoped that the self magnetic field of the high current would produce a constriction of the spark channel, and that this would so increase the voltage across the spark, and reduce the heat losses from it, that much higher temperatures, and almost 100% ionization of the gas would result. It was realized that the conditions necessary for efficient thermo-nuclear reactions could hardly be reached in a spark. In view, however, of the uncertainty of the shape of the positive ion energy distribution, especially for high energies (although deviations from a Maxwellian distribution in the present discharge conditions are extremely unlikely due to interaction effects considered by Cahn (1949)) and the unknown effect of high self fields on high pressure discharges, it was considered worthwhile to attempt the experiment.

The high current generator, a description of which will be published in full, consisted of a bank of capacitors of a total capacitance of $58.5 \mu\text{F}$, charged to a voltage of 25 kV, and then discharged in parallel through a circuit of low impedance. A damped oscillatory discharge was obtained in which the current had a maximum value of 285 kA, and a period of $29 \mu\text{sec}$. The tungsten electrodes of the spark gap were 2 cm apart, and were enclosed in a spark chamber containing deuterium at atmospheric pressure.

It would have been impracticable to have detected the emission of protons from the reaction, since the detector would then have been inside the chamber and so exposed to the heat and the shock wave from the spark. Attention was therefore concentrated on the detection of neutrons from the reaction. Since the yield was expected to be extremely small, a very sensitive detector was required. One using a Szilard-Chalmers reaction in potassium permanganate, and a technique developed by Rotblat (1941) were chosen, since neutrons could be detected over a large solid angle and the effect of many sparks integrated. Also unlike counter methods, it was free from problems caused by the large electrical interference from the spark.

The volume of 22 litres of an almost saturated solution of potassium permanganate was contained in glass vessels placed around the spark chamber, so that a solid angle of approximately 2π radians was subtended by the detector at the spark. A calibration with a Ra-Be neutron source showed that a total of 3×10^5 neutrons incident upon the detector produced a manganese activity which gave a counting rate equal to the background count.

A blank experiment, performed with no neutron sources in the vicinity, showed that the activity of the manganese increased the background by 25%. This activity was attributed to cosmic ray neutrons.

A total of 100 sparks, each carrying a peak current of 285 kA were passed through the deuterium in the space of 1 hour. No activity in excess of that in the blank experiment was detectable. The total number of neutrons emitted by the sparks must therefore have been less than

6×10^5 . Assuming a Maxwellian distribution of velocities amongst the ions, the ion temperature in the spark channel must have been less than 10^6°C .

Measurements are now being made by Allen (1950) on the current densities and energy dissipation in high current sparks in various gases and experimental conditions. Recent relevant work, theoretical and experimental, on the effect of self magnetic fields of discharges has been carried out by Thonemann and Cowhig (1950).

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Film Transfer in Helium II below 1°K

By E. AMBLER and N. KURTI
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[Received December 20, 1951]

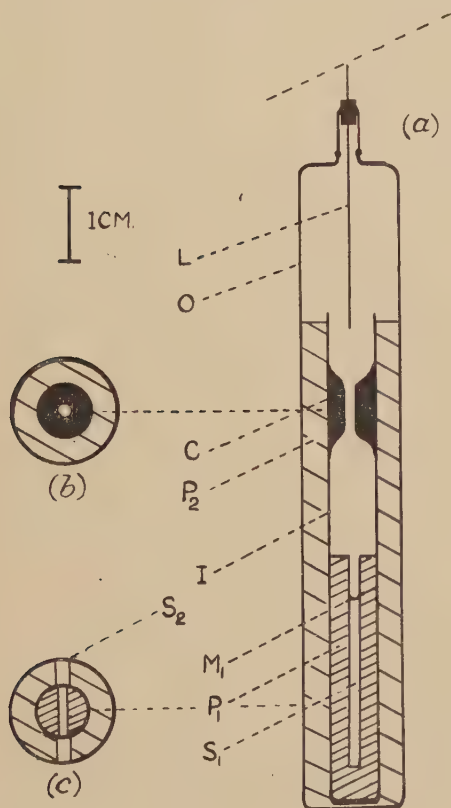
WE have recently carried out experiments on the rate of transfer in the helium II film at temperatures down to about 0.15°K and although the work is still in its preliminary stages it seemed worth while to report briefly not only the results obtained but also the experimental technique used which has enabled visual observations to be made on liquid helium in the range of temperature attained by the magnetic cooling method. From our experience we believe that measurements on other characteristic properties of helium II such as film thickness and viscosity can be extended into this range of temperature without too much difficulty.

The transfer rate has hitherto only been measured down to about 1°K , that is a reduced temperature of $T/T_\lambda = 0.4$. While most determinations have indicated a temperature variation roughly proportional to $[1 - (T/T_\lambda)^\sigma]$ where $\sigma \approx 6$ (Mendelssohn and White 1950), and a corresponding flattening of the transfer-vs-temperature curve below about 1.5°K we thought it worth while to extend these determinations to cover lower temperatures. Quite apart from the desirability, on general ground, of testing this law to smaller values of the reduced temperature, the range below about 0.7°K is physically, in one respect, different from the higher temperature range inasmuch as the vapour pressure of helium becomes so small (e.g. $1.6 \times 10^{-5} \text{ mm Hg}$ at 0.5°K)

* Communicated by the Authors.

that any possible influence of the vapour phase on the transfer rate by evaporation and recondensation or by heat conduction must become negligible (Kurti and Simon 1938). It should also be mentioned in this connection that the velocity of second sound increases sharply at about 0.5°K (Atkins and Osborne 1950) and the temperature variation of specific heat (Hull, Wilkinson and Wilks 1951, Kramers 1951) and of the heat conductivity (de Klerk 1946) changes fairly abruptly at about 0.6°K .

Fig. 1



The method of cooling liquid helium in an 'open' container by adiabatic demagnetization given by Hudson, Hunt and Kurti (1949) was used in these experiments. The container used for measuring the transfer rate is shown in fig. 1. It is suspended by means of nylon threads in the vacuum jacket of a glass demagnetization cryostat (not shown) containing liquid helium at about 1°K . The right-hand side diagram (a) is a longitudinal section and the two left-hand side diagrams (b) and (c) are cross sections. Film transfer takes place from the inner glass beaker I to the outer glass vessel O and is limited by the constriction C, 1.2 mm

diameter and 5 mm long in the upper part of I. The inner beaker contains a compressed pill P_1 of manganous ammonium sulphate in the form of a cylinder split in half longitudinally by a 1 mm wide slit S_1 . The space between I and O is filled with a closely fitting annulus made from a pill of the same salt having a 1.5 mm wide slit S_2 in the lower half in line with the slit S_1 . In order to be sure that the constriction C is at the same temperature as the salt, P_2 completely surrounds I over this portion of tube and is stuck to it by means of a cold setting plastic. Helium may be condensed into I through the fine capillary L. A further pill attached to L above the transfer vessel acts as a thermal shield (Hudson, Hunt and Kurti 1949).

Table 1

Run No.	Temperature °K	Transfer rate (cm ³ /cm/sec)
1	0.15	15.9×10^{-5}
	0.17	15.9
	0.17	15.9
	1.03	12.6
	1.23	11.0
	1.66	10.2
	2.02	4.8
2	0.20	15.8
	0.99	10.9
	1.20	10.8
3	0.16	16.4
	0.5	14.4
	1.42	12.2
	1.03	12.3

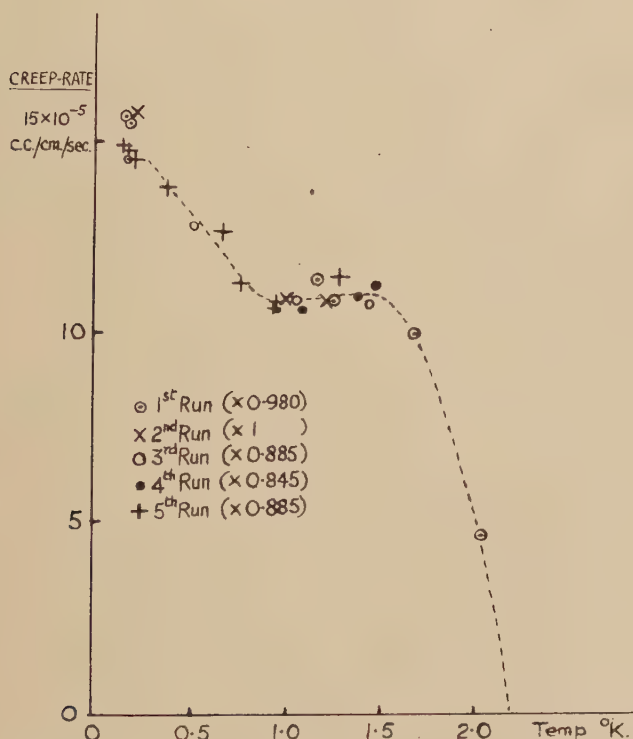
Run No.	Temperature °K	Transfer rate (cm ³ /cm/sec)
4	1.46	13.4×10^{-5}
	1.39	13.0
	1.07	12.6
	0.93	12.6
5	0.20	16.4
	0.14	16.9
	0.75	12.8
	0.93	12.1
	0.16	16.7
	0.37	15.7
	0.63	14.4
	0.93	12.2
	1.28	13.0

The demagnetization cryostat was enclosed in a vacuum jacket surrounded by a Dewar vessel containing liquid hydrogen. Apart from vertical slits through which the illumination and viewing of the helium took place, the glass walls of the cryostat, vacuum jacket and Dewar vessel were silvered to minimize the radiation falling on the pills. The slit S_1 was illuminated by means of a narrow parallel beam of light from a small mercury lamp, suitable filters ensuring that only the green light was used: stops limited the cross section of the beam to about the size of the slit S_1 . In this way we minimized the amount of light falling actually on the salt, a precaution made necessary by the apparently strong absorption by the salt of visible light, which, as was shown in preliminary tests would have given a prohibitively large heating up rate. For the same reason the slit was only illuminated during the actual time

of observation which, by practice, was reduced to a few seconds. In these conditions the transfer vessel remained below 1°K for about half an hour, corresponding to an average heat influx of 2 000 ergs/min.

The experimental procedure was as follows. A calculated quantity of helium was condensed into the inner beaker so that after demagnetization, which, from an initial H/T of 10 kilo-oersted/degree led to about 0.15°K , the meniscus M_1 would be near the top of the slit S_1 . After demagnetization the position of M_1 was determined every two or three minutes and, during the intervening periods, the susceptibility of the salt from which the

Fig. 2



absolute temperature could be estimated (Cooke 1949) was measured by the ballistic method. From the rate of fall of M_1 and the free cross-section of I the creep-rate through C was calculated.*

The results are given in table 1; it will be seen that the rates above 1°K are somewhat higher than those usually observed for glass (Daunt and Mendelssohn 1939), and there are differences of up to 15% between different runs. This may have been caused by slight contamination of the constriction in the inner beaker (Bowers and Mendelssohn 1950). For the purpose of comparison each set of results from different runs

* To a transfer-rate of $10 \times 10^{-5} \text{ cm}^3/\text{cm}/\text{sec}$ corresponds a meniscus movement of 0.30 mm/min.

was 'normalized' (by multiplication by a constant factor) so as to give identical transfer rates at 1.2°K . The results so obtained are plotted in fig. 2 where it will be seen that after the flat portion between 1.5°K and 0.8°K the transfer-rate shows a further increase which amounts to about 30% at 0.2°K .

We have considered whether this increase may be due to some spurious effect such as a preferential heating of the inner beaker caused by the illumination and consequent distillation into the outer vessel, but the effect seems to be too large to be explained in this way. Before drawing any conclusions as to the fundamental implications of this result we should prefer to have more measurements made on the film both on other surfaces and under different experimental conditions at these very low temperatures. It is worth noting, however, that the results of Burge and Jackson (1951) do show an increasing film thickness as the temperature falls below 1.4°K .

We wish to express our grateful thanks to Professor F. E. Simon for his stimulating interest, to Mr. W. E. Gardner for his untiring help with the experiments and to the Department of Scientific and Industrial Research for E. A.'s maintenance grant.

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Fine Structure in the U238 Decay

By BARBARA ZAJAC

with Appendix by N. MILLER

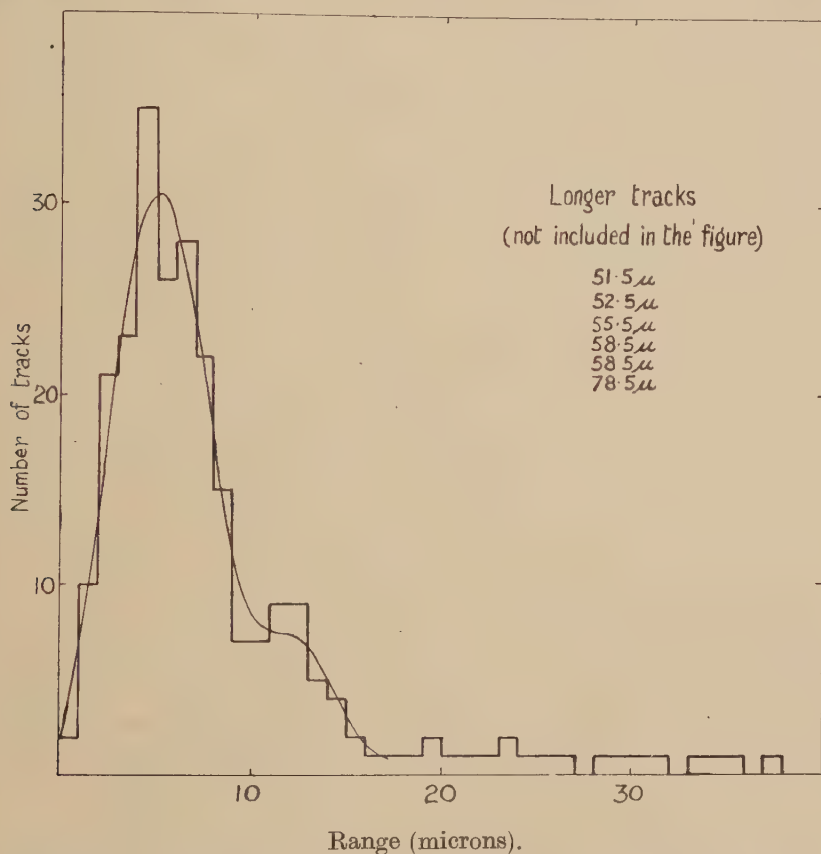
Department of Natural Philosophy, University of Edinburgh*

[Received December 27, 1951]

ILFORD G5 nuclear emulsions have been impregnated with natural uranium in solution as the complex ammonium citrate, adjusted to pH 8. Prior to conversion into citrate the uranium was freed from UX_1 and other decay products and impurities by extraction of the crystalline nitrate into ether, followed by washing of the ethereal solution with a little water. The plates were exposed for 72.5 hours.

* Communicated by Professor N. Feather.

The developed plates were examined for the occurrence of electron tracks in association with the tracks of the α -particles of the three isotopes of natural uranium. A statistical survey showed that such electron tracks were associated with $29.1 \pm 1.3\%$ of all uranium α -particle tracks. Measurements were made on 776 α -particle tracks to give distribution of ranges for 'all' α -particles, and on the tracks of 635 α -particles having associated electrons. The two range distributions were very similar.



Distribution of ranges of electrons associated with α -particles of ranges smaller than 17.55μ .

In both two main peaks occurred at about 16.5μ and 19.5μ emulsion range and there were signs of a small (incompletely resolved) peak at about 17.5μ range due to the 4.4 mev α -particles of U235 (Baldinger and Huber 1949, Ghiorso 1951). The similarity of the two α -particle range distributions showed conclusively that secondary electrons follow the α -disintegration of U238 as they do that of U234.

The results of measurement of electron tracks associated with U234 α -disintegrations were in general agreement with those reported previously by Teillac (1950).

The diagram represents the histogram (and the corresponding smoothed curve) of ranges of electrons associated with α -particles having ranges smaller than 17.55μ , i.e. with the α -particles of U238 and U235. The total number of events represented in the figure is 232. A careful estimation showed that less than a quarter of these could involve UX_1 and UY disintegrations, following the U238 and U235 disintegrations respectively, or internal conversion of the known γ -rays of U235. The high energy tail of the distribution might well belong to these events, but the main part of the low-energy component, that is fully three-quarters of the electrons, must be a feature of the U238 decay.

Sixteen events are included in which more than one (two, three or four) electrons were associated with a single α -particle. All of these could be interpreted in terms of subsequent UX_1 and UX_2 (Heershap, Ong Ping Hok and Sizoo 1950), or possibly UY , disintegrations.

The main component of the electron range distribution shows two maxima at 5.2μ and 12μ range, corresponding to electron energies of 27.5 and 45 kev (Zajac and Ross 1949) respectively. Electrons having these two energies may be interpreted as L and $(M+N)$ internal conversion electrons of a γ -ray of about 48 kev energy.

A calculation taking into account all the facts leads to an estimate that $23 \pm 3\%$ of U238 α -disintegrations are associated with an L or $(M+N)$ internal conversion transition of 48 kev total energy.

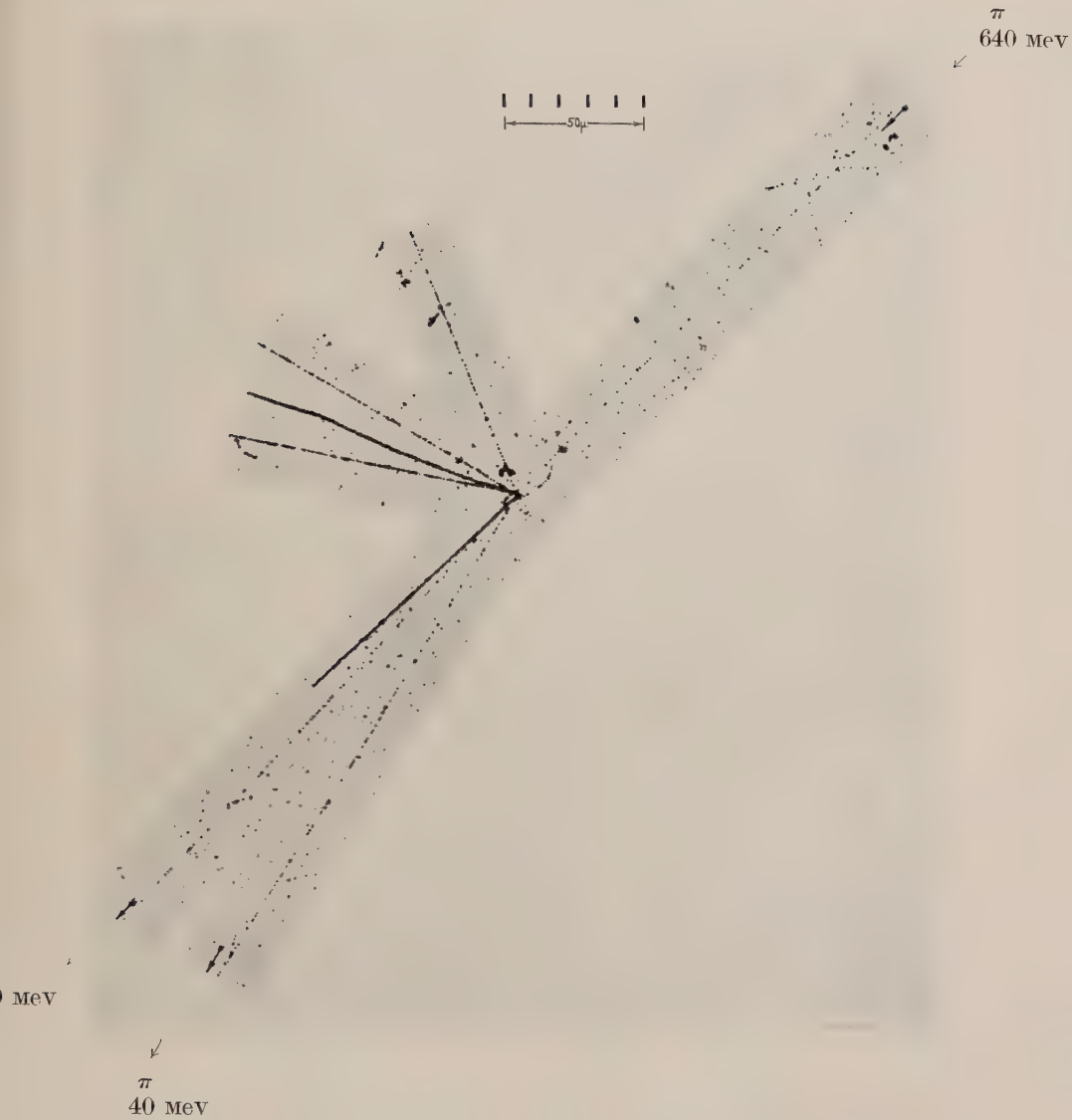
APPENDIX

For emulsions of this type to remain fully electron sensitive it is essential that any impregnations be carried out in neutral solutions (Picciotto 1949). If impregnations with heavy elements are desired, this usually means in practice that a complexing agent has to be added. Citrate ions are found to be satisfactory for a wide range of heavy elements, and also prevent the precipitation of carbonate within the emulsion during development when barium carriers are added.

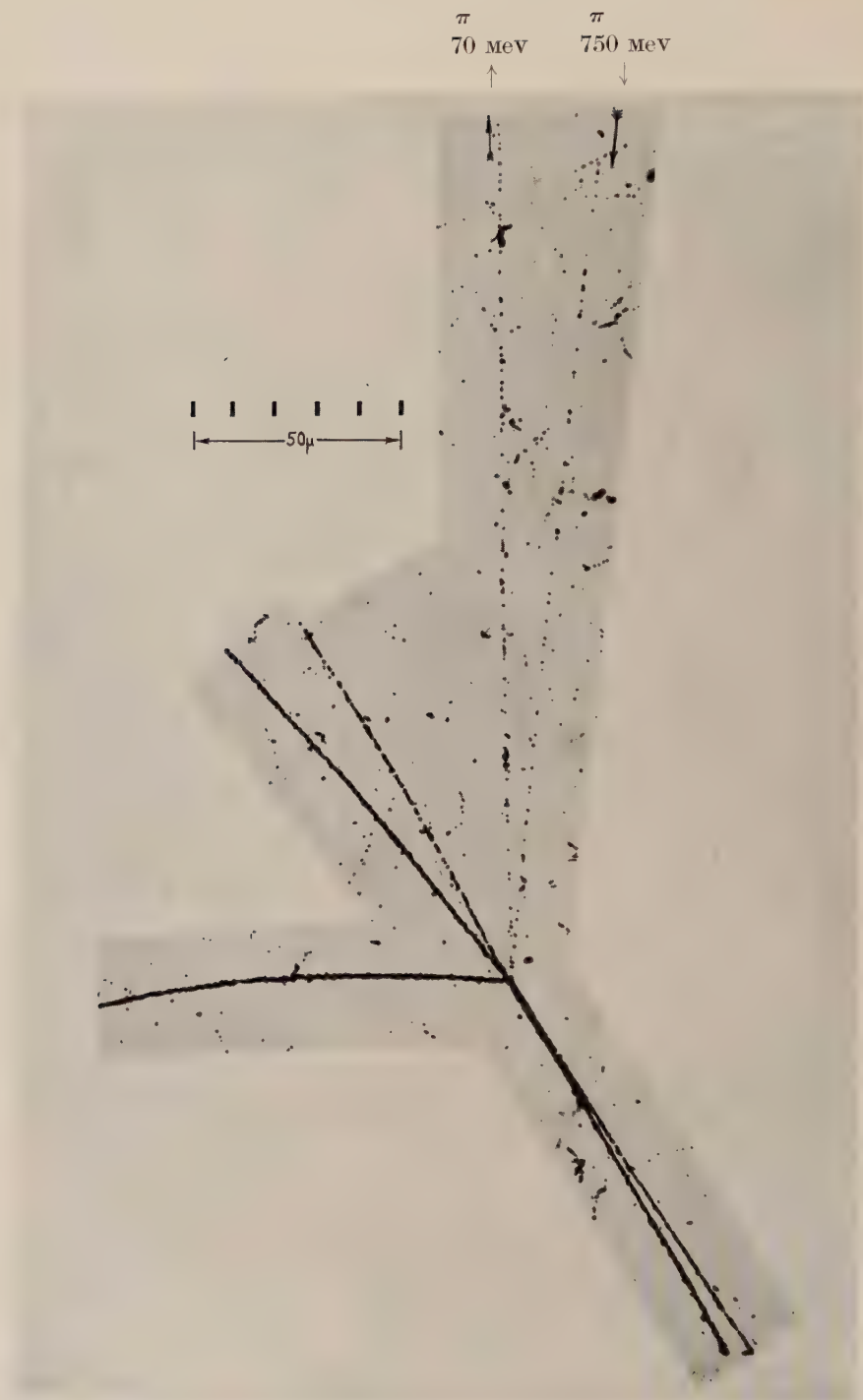
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[The Editors do not hold themselves responsible for the views expressed by their correspondents.]



A disintegration initiated by a π -meson of kinetic energy 640 mev from which a π -meson of energy 40 mev is ejected, together with a proton of 380 mev.



A disintegration initiated by a π -meson of 750 mev kinetic energy from which a π -meson of 70 mev energy is ejected at an angle of 167° to the primary particle.

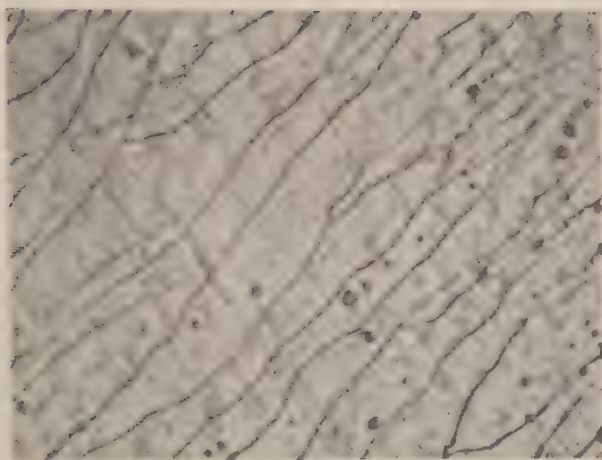


Fig. 1

$\times 375$



Fig. 2

$\times 280$

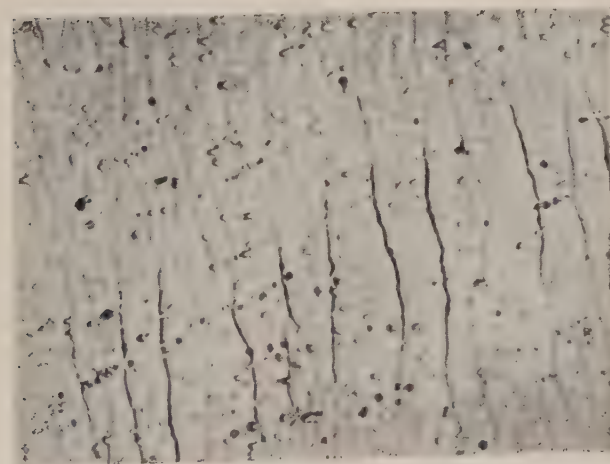


Fig. 3

$\times 280$

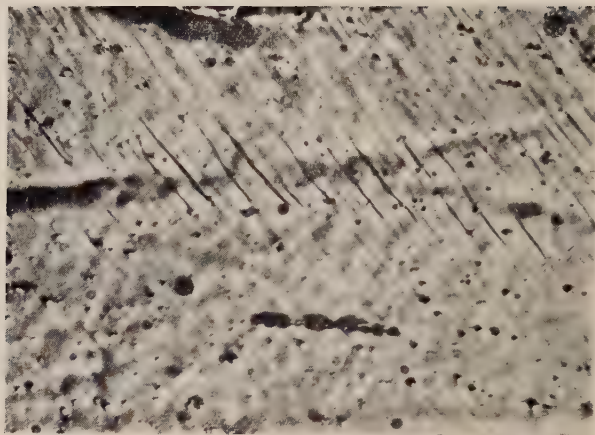


Fig. 4

$\times 280$

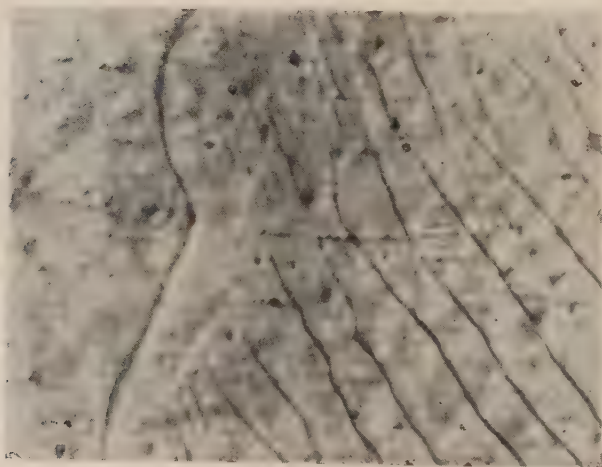


Fig. 5

$\times 375$

O B I T U A R Y

ALLAN FERGUSON, 1880-1951 :

AN APPRECIATION

BY H. R. ROBINSON

ALLAN HITCHEN FERGUSON was born at Entwistle in May 1880, and his boyhood was spent in and near mid-Lancashire. He got his first systematic instruction in science at the Harris Institute, Preston, where he began his studies for a London External degree. In 1902 he entered the University College of North Wales, Bangor, as an Exhibitioner, and in due course took the M.A. degree of the University of Wales in Mathematics, and the London B.Sc. degree in Physics. He obtained the London D.Sc. degree in 1915.

From 1905 to 1919 he served on the physics staff at Bangor ; during the first world war he was also for some time in charge of the applied mathematics courses, and acted as lecturer in electrical engineering. During his later years in Bangor, he devoted much attention to hydro-electric problems, with particular reference to the possibility of developing water-power resources in North Wales. He left Bangor in 1919, and was for two years lecturer in physics at the Manchester Municipal College of Technology (in the Faculty of Technology of Manchester University).

From Manchester he came in 1921 to join the staff of East London College (now Queen Mary College) in the University of London, as second-in-command to Professor C. H. Lees, F.R.S. He was made Assistant Professor of Physics in 1934, and he remained in the service of Queen Mary College until his retirement in 1945. While the College was in Cambridge in 1939-45, he assisted the depleted staff of the Cavendish Laboratory by lecturing to Cambridge Tripos candidates, in addition to his work for London students.

This bare list of his teaching appointments, possibly of no great interest in itself, does lead to one point which needs more than a passing mention. This is that for the most part his teaching life was spent in departments that were far from lavishly staffed, and over the years he must have lectured at one time or another in all branches of classical physics, and at all stages of university degree courses. His special interests lay mainly in general physics and thermodynamics, but he was always ready to take over at the shortest notice an advanced honours course in optics or electricity—and he did it all supremely well. He was one of the best university teachers in the country, and there are today scores of practising scientists who hold him and his teaching in grateful memory.

At Queen Mary College he took a large share in organizing the teaching of physics and in supervising postgraduate research. He was moreover an outstanding figure in the general life of the College ; he enjoyed to a

high degree the confidence of his colleagues on the teaching staff, and from 1934 to 1944 he represented the Academic Board on the Governing Body. In 1946 the College fittingly recognized his many services by the award of the highest honour in its gift, a Fellowship.

He had an unusually sound and comprehensive knowledge of physics in general, and of thermodynamics and the properties of matter in particular. Indeed, as early as 1906, G. H. Bryan, in the preface of his treatise on Thermodynamics (published Teubner, 1907) acknowledged the help given to him in its preparation by "Mr. Ferguson, B.Sc.". Ferguson's own research work lay mainly in the fields of the physics of liquids and the difficult borderland between pure physics and physical chemistry. He was a recognized authority on surface tension, and he contributed extensively to the study of electrolytic conduction. He also, during the five or six years which preceded the outbreak of the second world war, gave much time to perfecting and applying an ingenious and powerful electrical method which he had devised for measuring the specific heats of liquids and their temperature variations. All his research work was characterized by great elegance and thoroughness; he excelled in devising methods which were at once simple and accurate, and his published descriptions of them were models of clear and graceful exposition. Simplified forms of some of his original experiments—notably one for the determination of the surface tensions of liquids available only in very small quantities—have been incorporated into the routine of undergraduate laboratory courses, and they never fail to impress and interest the better students.

His contributions to the advancement of physics were, however, far from being confined to work done within the walls of the laboratory. He was an active member of a number of scientific societies, and he rendered valuable services to these societies, and through them to the general body of physicists. This was particularly evident in his work for the Physical Society and the British Association for the Advancement of Science. He joined the Physical Society when he came to London in 1921, and became a member of its Council four years later. In 1928 he accepted the office of Honorary Papers Secretary, which he retained until he was elected to the Presidency of the Society in 1938. It was about half-way through his period as Papers Secretary that the publication was begun of the Physical Society's annual *Reports on Progress in Physics*, and it is no secret that Ferguson, if not indeed the "onlie begetter" of the series, was mainly responsible for its inception. He personally edited the first five volumes (1934-1938), and raised the series at once to the rank of a scientific periodical of international repute.

The original intention was "to present articles which shall discuss the latest developments in physical science, and which shall at once make clear to the non-specialist the meaning and extent of these advances, and provide the researcher in the particular field under discussion with a technical résumé helpful to him in his own work". The need for a

series of authoritative summaries of this kind was obvious in 1934, for many branches of physics were then proliferating with bewildering speed; the main difficulties to be expected in fulfilling the plan were equally obvious—that of making the best choice of fields for reporting, and that of persuading active workers in those fields to help with the reports. Ferguson succeeded in the first place through his wide outlook and clear vision as a physicist, and in the second place quite simply because he was so widely known and well liked in scientific circles that he found it easy to get the right kind of help. The *Reports* were originally planned as no more than a service to the Fellows of the Physical Society, and as Ferguson modestly wrote in 1938, "... it is by a happy chance that they interest a public outside the membership of the Society". It was, of course, through no chance at all that this happened, and the success of the *Reports*, and the added reputation they brought to the Physical Society, may well be regarded as a memorial to Ferguson's qualities as a physicist and his gift for getting the best out of others.

He was President of the Physical Society for the three years 1938–1941. He filled the presidential chair with distinction, and the Society owes much to his wise guidance during the critical early years of the war. From September 1939 onwards he was working in Cambridge, but the records of the Society show that he was regularly in the chair at its London meetings—not only during his Presidency, but also frequently as Acting President after his own term of office had expired. This was done in spite of increasing physical disabilities, and especially impaired vision, which made travelling under war conditions very burdensome, and many of his friends felt and said that he was undertaking too much. He had his reward in seeing the Society's activities vigorously maintained, under conditions in which more timid leadership would have cut them down.

He gave equally distinguished services to the British Association for the Advancement of Science. After being first a Secretary and then Recorder of Section A, he became an ordinary member of Council at the end of 1932, and from 1935 to 1946 he served as one of the two General Secretaries of the Association. He was President of Section A at the Blackpool meeting in 1936, when he took as the subject of his Sectional Address: "Trends in Modern Physics". It was a wise and witty address, and it is full of interest as a frank statement of the reactions of a philosophic mind, steeped in the physics of Kelvin and Maxwell, to the impact of a newer synthesis.

Ferguson also took a part in the activities of the Institute of Physics, serving as a member of the Board from 1928 to 1932, and again from 1934 to 1938, and as a Vice-President from 1939 to 1943. He was a member of the editorial board of *Endeavour* from its beginning in 1942, and for a short period in 1939–1940 he was an Advisory Editor to *Nature*.

He had frequently expressed to me his affection for the Philosophical Magazine, and I remember the pleasure with which he received, in 1937,

an invitation to join its Editorial Board. For some years he was Executive Editor, until increasing physical disabilities compelled him to retire. One of his last direct services to the Philosophical Magazine was the editing of the Special Commemoration Number of 1948 (*Natural Philosophy through the 18th Century, and Allied Topics*).

Another important task which he carried out after retiring from university teaching was that of advisory editor of the physics and engineering sections of the new Chambers's Encyclopaedia.

He was an excellent popular lecturer, and a good broadcaster on scientific and general topics. Many members of the Royal Institution will remember with pleasure a Friday Evening Discourse he delivered in 1934; his subject was "Surface Tension", and he showed (I think, for the first time at a public meeting) a beautiful slow-motion film of the formation of drops and splashes, which had been made with a high-speed camera taking nearly 2000 exposures a second. This development of high-speed cinematography had a special interest for him, and he lectured frequently on it, as in the British Association's Alexander Pedler Lecture (Cardiff, 1940).

To come now to the more personal side of his life and character: he was physically a very big man, standing several inches over six feet in height, and rather more than proportionally broad. In fact he tended to corpulence, and he would at times express a mild disdain for men who fell far short of the 20 stone which he affected to regard as the canonical weight for a physicist. He was, however, before illness overtook him, active and muscular, and in no way handicapped by his bulk.

He was one of the friendliest souls alive, and his habitual expression was one of lively good-humour—which largely accounts for the ease of his relations with colleagues and pupils. Even the most diffident students were quick to realise that this large, impressive figure was infinitely approachable and benign, and he was particularly effective as an oral examiner, in getting the best out of nervous candidates.

He was a voracious and omnivorous reader, and an incorrigible collector of books—especially old books. He had a very pleasant and commodious house in Bishop's Stortford, where he lived for the last 22 years. In this, as an energetic amateur carpenter of the rough-and-ready school, he fitted open shelves into every available alcove and corner, and he finally had a large book-room, shelf-lined, built as an annexe, but his shelving always lagged far behind his acquisitions. Books were stacked all over the house, in a way that would not have been allowed in any but a tolerant and thoroughly book-loving household. His tastes were catholic, but naturally with a bias towards old science. He made no claim to a systematic knowledge of the history of science, but he was extremely well-informed on many sections of it; an instance of this may be seen in his delightful "Introductory Survey" to the British Association Centenary volume on *London and the Advancement of Science* (1931). As an amateur historian of science, he had the advantage of a

solid background of literary and general history; he saw scientific discoveries not as isolated events, but in their relation to the general conditions of the times in which they were made.

Although he did not confine himself to it, the 18th was his favourite century in literature, and Samuel Johnson his chief literary hero. In fact I think he liked to dwell in imagination on resemblances, real and fancied, between himself and Johnson. There was at least one real difference between them, which was a blessing to his friends—Allan Ferguson was a patient and sympathetic listener, as well as a good talker, and he loved the give-and-take of round-the-table conversation. Although he had, for instance, an intimate knowledge of church architecture, he would never thrust the subject into a general conversation; and although, like nearly all men of his period, he had a store of favourite anecdotes—including, of course, a number of “tacklers’ tales” from his beloved Lancashire—he had none of the tiresome habits of the persistent raconteur; in brief, he talked for enjoyment, and not for effect.

In politics he was a Gladstonian Liberal, and for many years he was President of the Hertford Divisional Liberal Association; less publicly, he was in sympathy with many aspects of socialism, and in some purely personal matters he was healthily tinged with conservatism. A man of strong courage, he was a hater of war and an active supporter of the League of Nations Union; and the failures of the League appeared to weigh more heavily upon him than any of his personal tribulations.

He was above all things fortunate in his family life. He married in 1919 Nesta Thomas, daughter of the Vice-Principal of the Normal College, Bangor. She had already proved herself a botanist of distinction, and had successively held a research scholarship and a Fellowship of the University of Wales, and a Lectureship in Botany in the Royal Holloway College (University of London). She continued her botanical work after her marriage, undertaking part-time university and other teaching and examining at various periods, and she obtained her London D.Sc. in 1927—twelve years after her husband. Their only child, John, had an outstandingly brilliant record of achievement at school and at St. John’s College, Cambridge, and is now a Lecturer in Classics in the University of Durham. (He is, incidentally, the “John Ferguson, M.A., B.D.” who was co-author with his father of the introductory article in the Commemoration Number of the *Philosophical Magazine*, 1948.)

Ferguson took great pride and pleasure in the academic distinctions and activities of his wife and son; more than this, they were a powerful consolation to him at a time when he most needed comforting.

The story of the bodily afflictions of his later years, painful though it is, ought not to be entirely omitted; for it is also a story of high courage and continued activity under conditions that would have reduced most men to a sterile despair. His first serious troubles came in 1933 and 1934, when he was threatened with the possibility of total blindness. This was averted by skilled surgery, and although the resulting condition

was something of a handicap in outdoor movement, he did not let it restrict his literary and scientific activity. Early in the second world war, however, there came signs of graver organic disease; he was affected by a serious and progressive form of paralysis, followed by other painful complications. I saw much of him in the earlier years of this long illness; we shared a room in the Cavendish Laboratory during a period when he was unable to lecture without the aid of powerful drugs. Nevertheless, he insisted on continuing his work, and in spite of all he remained the sweetest of companions. He was not, of course, infinitely tolerant of all things, and it was always possible to draw a shower of sparks from him by such simple devices as claiming an Englishman's right to close a sentence with a preposition; but I only once heard him grumble about his own tribulations, and that was to make a purely jocular complaint that he could no longer compete on equal terms in spotting the bargains on David's bookstall. His intellect remained completely unclouded, and even those who met him for the first time in this darkening period were deeply impressed by his personality and his mental power. For all who enjoyed his friendship—and they are very many—his pleasant voices, his nightingales, will be awake while memory lasts.

H. R. R.

The Directors of Taylor & Francis Ltd., proprietors of the *Philosophical Magazine*, would like to add a note telling of their indebtedness to Professor Allan Ferguson. He took over the Acting Editorship of this Journal on the death of Professor A. W. Porter in 1937.

The war period was one of great stress for the Magazine. The difficulty of obtaining papers of the high standard expected of the *Philosophical Magazine* and the almost overwhelming difficulties of publication made Professor Ferguson's task a very arduous one. It will be admitted that he overcame the various troubles extremely well, especially when it is remembered how badly handicapped he was with ill health during the last few years of his editorship.

In addition to his work on the *Philosophical Magazine* his advice in many directions on policy was invaluable.